



Introduction to Quantitative Geology

Lecturer: Ann-Kathrin Maier

Week 5 – Rocks and ice as viscous materials

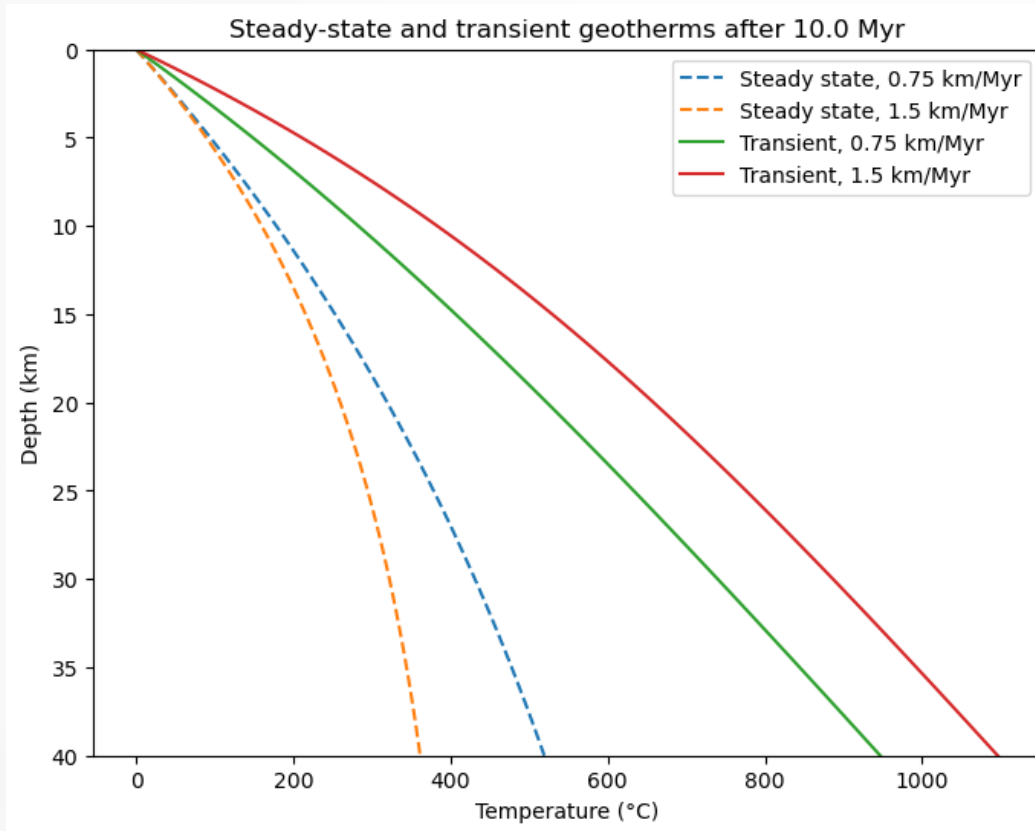


Last week

- **Part 1: Advection**
 - The advection equation
 - Advection in geological processes
- **Part 2: Erosion, sedimentation, heat transfer**
 - Some definitions
 - What happens with the geothermal gradient?



Ex. 4 Optional question



- **Steady-state boundary conditions:**
 - A constant surface temperature of 0 °C
 - A constant thermal gradient at the surface



This week

- **Rocks and ice as fluids**
- **Viscosity**
 - Definition
 - Viscous flow in a channel
 - Effects of temperature
 - Non-linear viscosity
- **Case study: Glaciers**

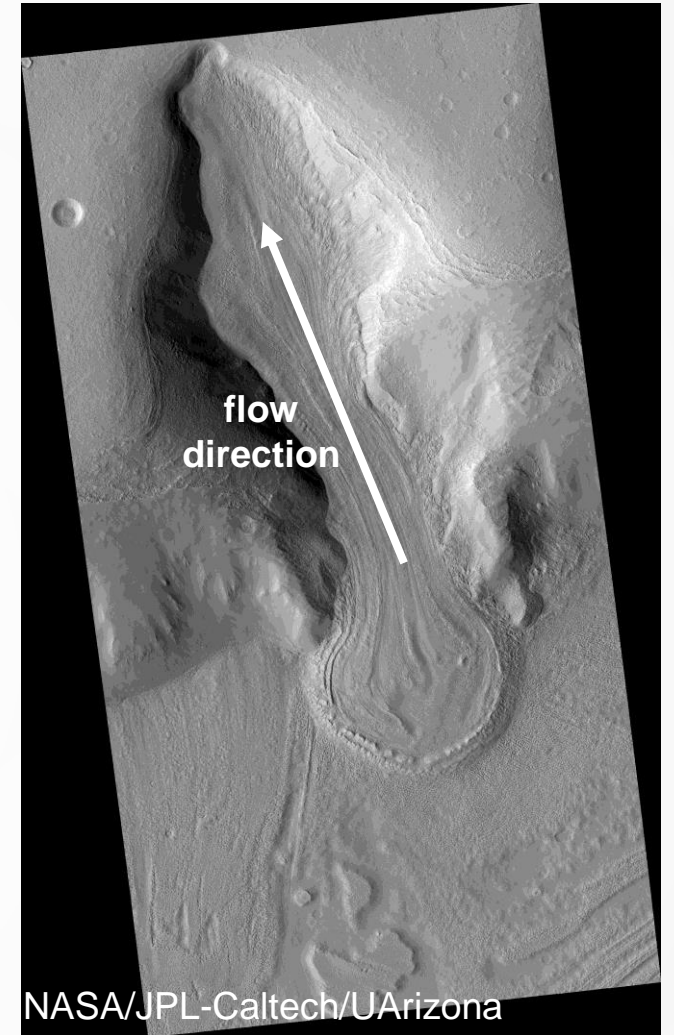


Examples of viscous flow: Glaciers



Riggs Glacier, Alaska, USA

Glaciers
flow under
their own
weight

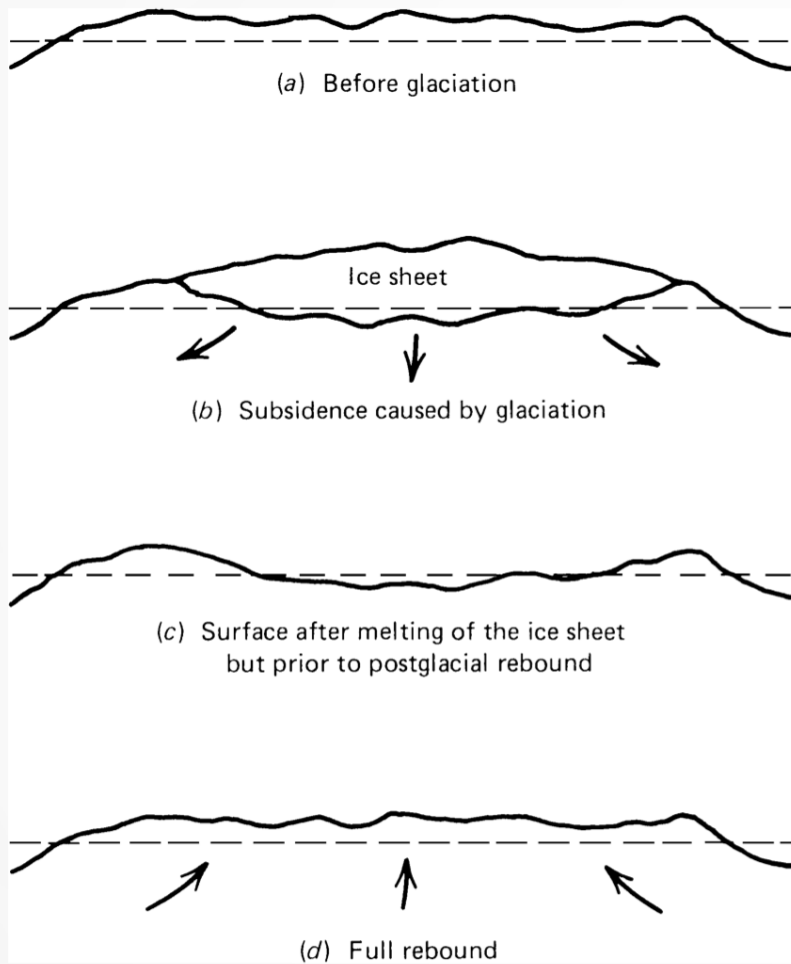


NASA/JPL-Caltech/UArizona

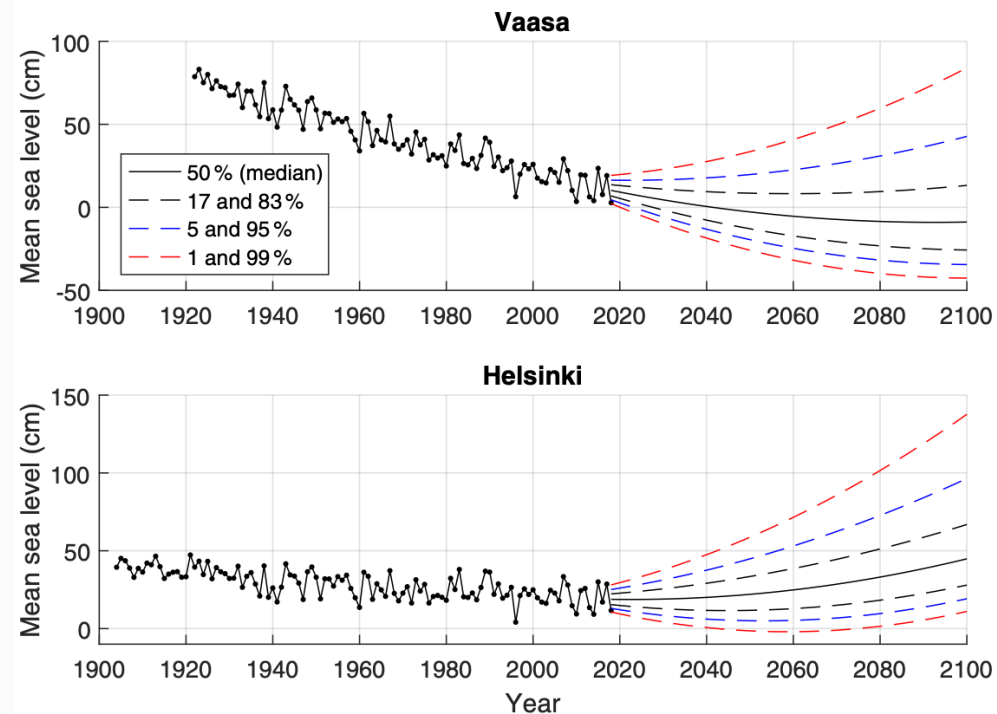
**HiRISE image of glacial-
like form on Mars**



Glacial isostatic adjustment



Turcotte and Schubert, 2014

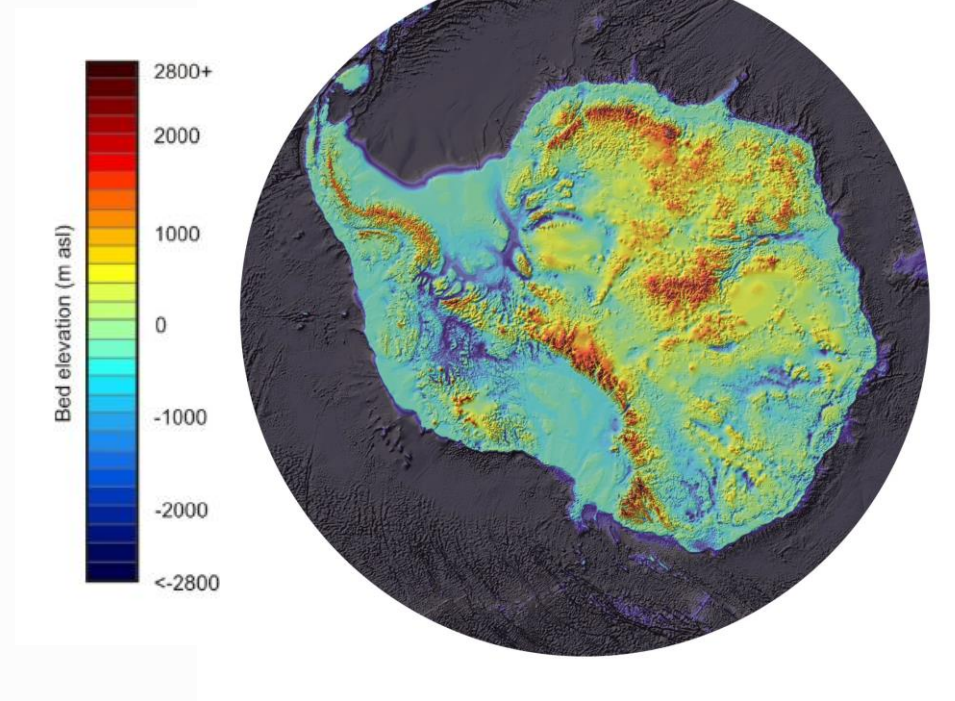
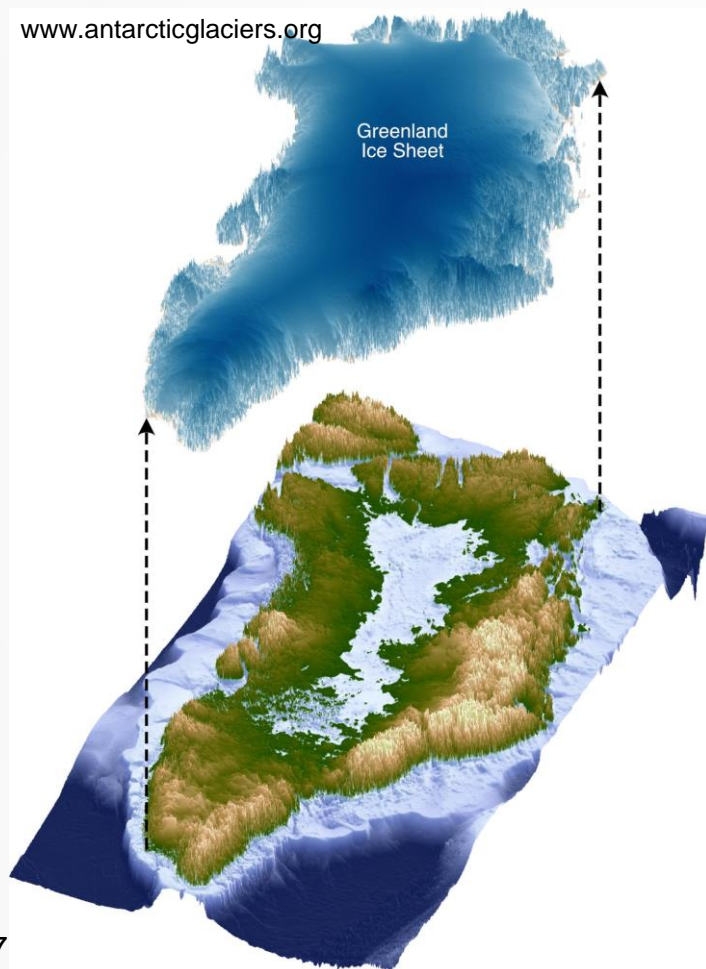


Pellikka et al., 2023

Surface uplift due to glacial isostatic rebound is controlled by **flow of the underlying asthenosphere.**



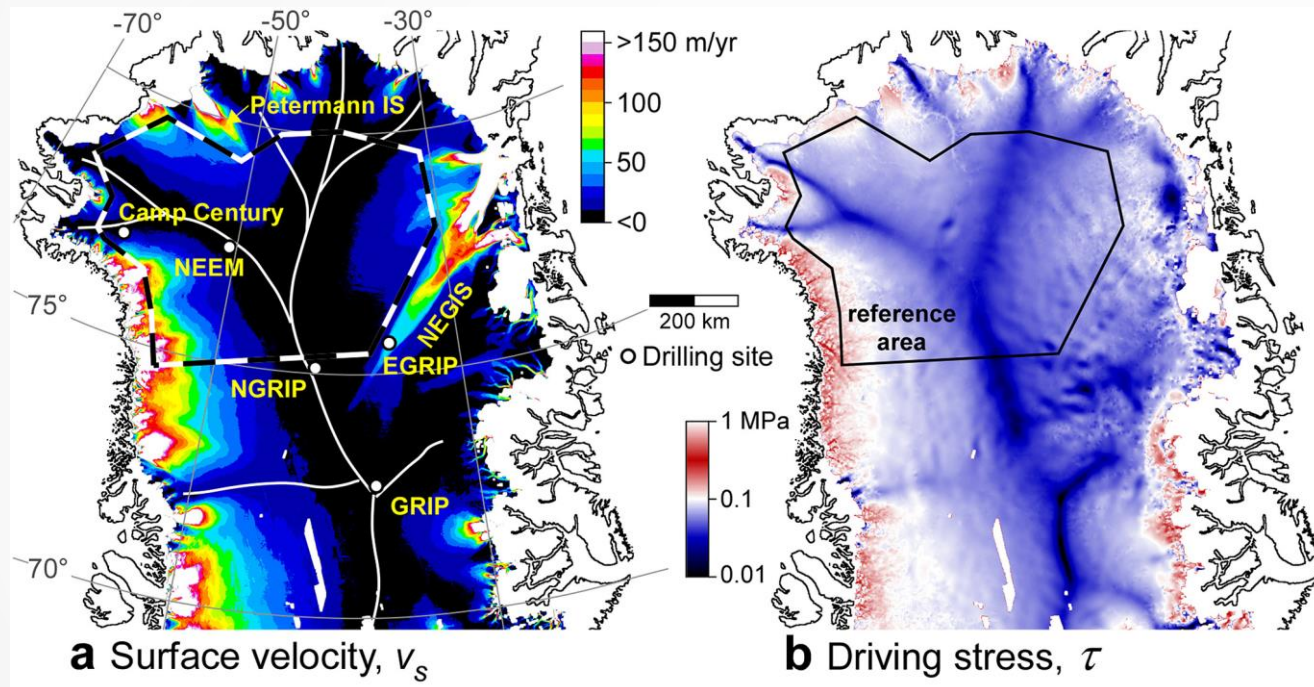
Beneath the ice



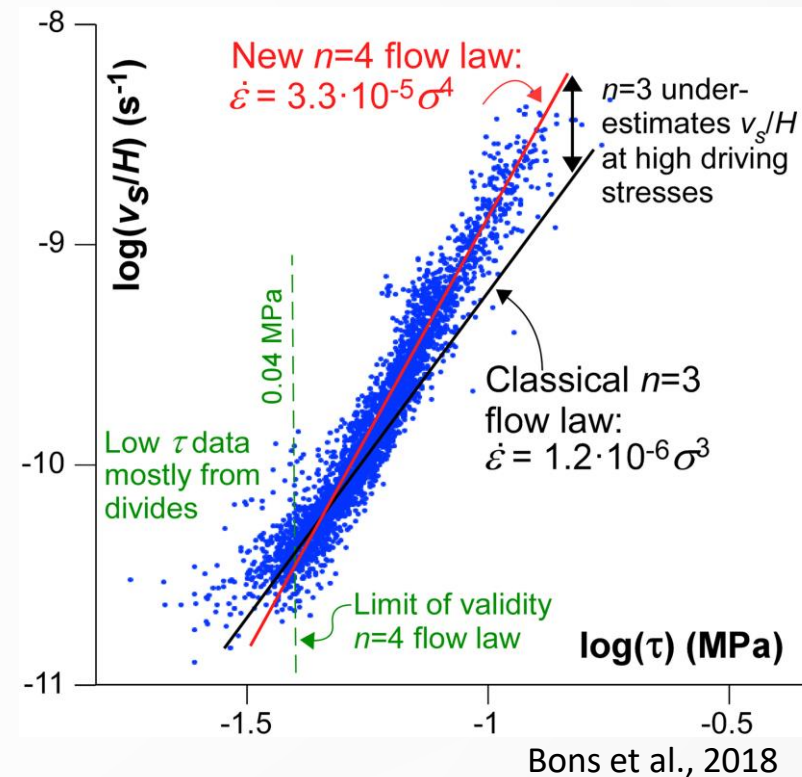
Parts of Greenland and Antarctica lie below sea level.



How do we know how rocks and ice flow?



Bons et al., 2018



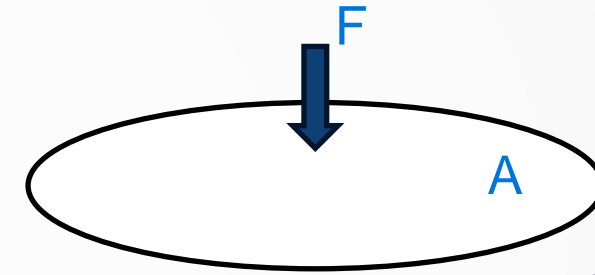
Flow laws.



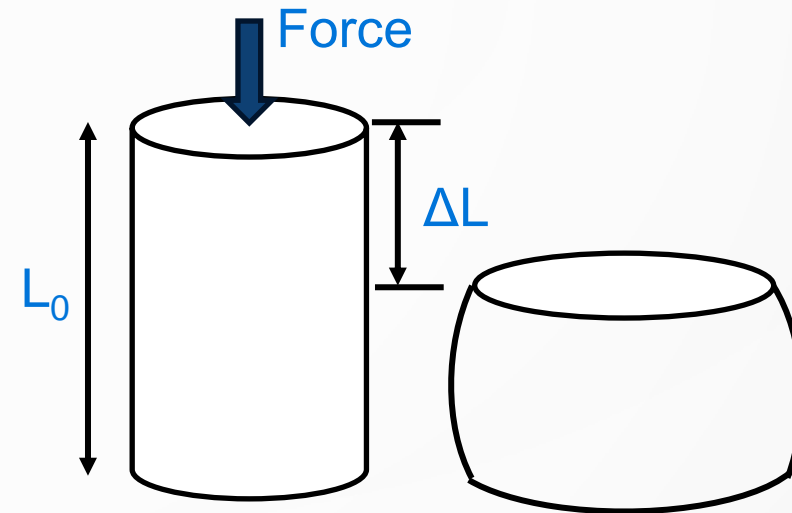
Rocks and ice as fluids

- **But first: stress and strain!**
 - **Stress (τ)** is the force divided by area
 - **Strain (ϵ)** is a measure of change in shape
 - **Strain rate ($\dot{\epsilon}$)** is the rate of deformation

$$\dot{\epsilon} = d\epsilon/dt$$



$$\tau = F/A$$



$$\epsilon = \Delta L/L_0$$



What is a fluid?

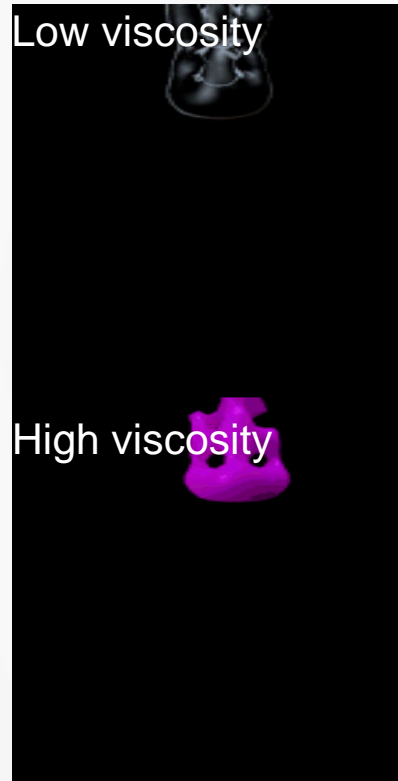
- **Fluid:** Any material that flows in response to an applied stress
 - Deformation is continuous
 - Stress is proportional to strain rate

$$\tau \propto \frac{du}{dz}$$

where τ is the **shear stress**, du/dz is the **velocity gradient** (equivalent to strain rate) and u is the **velocity in the x -direction**



Definition: Viscosity



<http://en.wikipedia.org>

- We can think of **viscosity** as a resistance to flow
Higher viscosity → more resistant to flow and vice versa
- Stress proportional to strain rate → Constant of proportionality **η** is known as the **dynamic viscosity**, or often simply viscosity

$$\tau = \eta \frac{du}{dz} \quad (1-D)$$

- Viscosity has units of **Pa s** (Pascal seconds) or $\text{kg m}^{-1} \text{s}^{-1}$
- Kinematic viscosity and bulk viscosity are different from dynamic viscosity



Approximate viscosities of common materials

Material	Viscosity [Pa s]
Air	10^{-5}
Water	10^{-3}
Honey	10^1
Basaltic lava	10^3
Ice	10^{10}
Rhyolite lava	10^{12}
Rock salt	10^{17}
Granite	10^{20}



A honey dipper works because of the viscosity of honey

- Viscosity of natural materials is hugely variable
 - Range of almost 20 orders of magnitude for rocks and lava



Newtonian materials



$$\tau = \eta \frac{du}{dz}$$

- A Newtonian material has a linear relationship between shear stress and strain rate
- In other words, η is a constant value that does not depend on the stress state or flow velocity
- Air, water and thin motor oil are practically Newtonian fluids 😊
- Rocks rarely deform as Newtonian fluids 😞

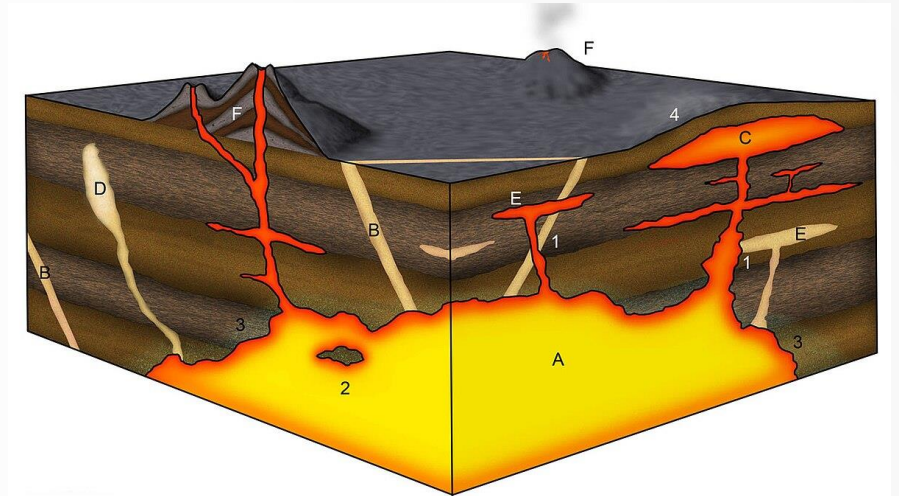


Describing the flow of fluids



https://commons.wikimedia.org/wiki/File:Grosses_Aletsch_gletscher_3196.JPG

in geology



https://commons.wikimedia.org/wiki/File:Igneous_structures.jpg



[https://commons.wikimedia.org/wiki/File:Yellowstone_National_Park_\(WY,_USA\)_Old_Faithful_Geyser_-_2022_-_2599.jpg](https://commons.wikimedia.org/wiki/File:Yellowstone_National_Park_(WY,_USA)_Old_Faithful_Geyser_-_2022_-_2599.jpg)



1D linear viscous channel flow

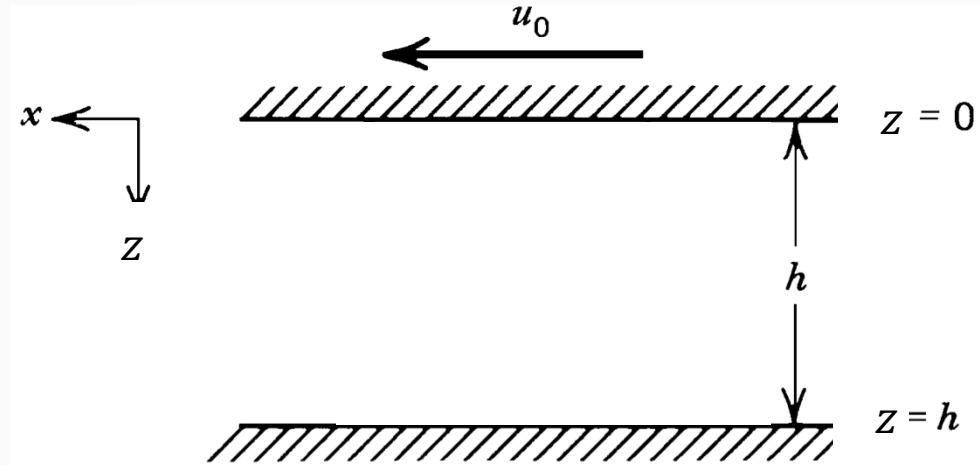


Fig. 6.2a, Turcotte and Schubert, 2014

- The general solution for the 1-D velocity of a fluid across a channel with boundary conditions (1) $u = 0$ at $z = h$ and (2) $u = u_0$ at $z = 0$ is

$$u = \frac{1}{2\eta} \frac{dp}{dx} (z^2 - hz) - \frac{u_0 z}{h} + u_0$$

where dp/dx is the applied pressure gradient



Styles of linear viscous flow: Couette flow

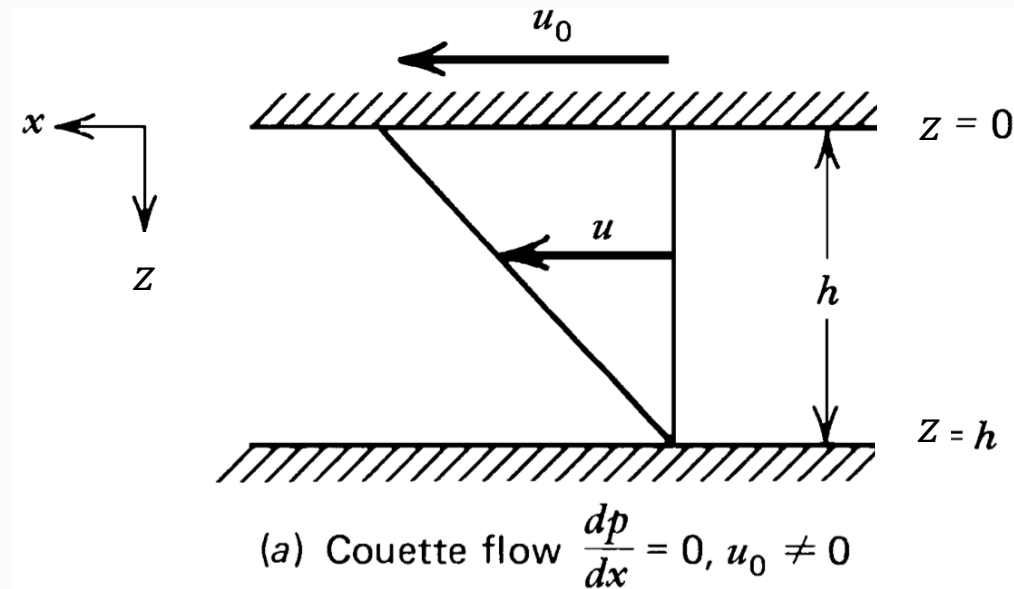


Fig. 6.2a, Turcotte and Schubert, 2014

- **Couette flow** occurs when there is (1) a difference in velocity between the channel boundaries and (2) effectively no pressure gradient



Couette flow solution

- If we assume $dp/dx = 0$,

$$u = \frac{1}{2\eta} \frac{dp}{dx} (z^2 - hz) - \frac{u_0 z}{h} + u_0$$

reduces to

$$u = u_0 \left(1 - \frac{z}{h} \right)$$

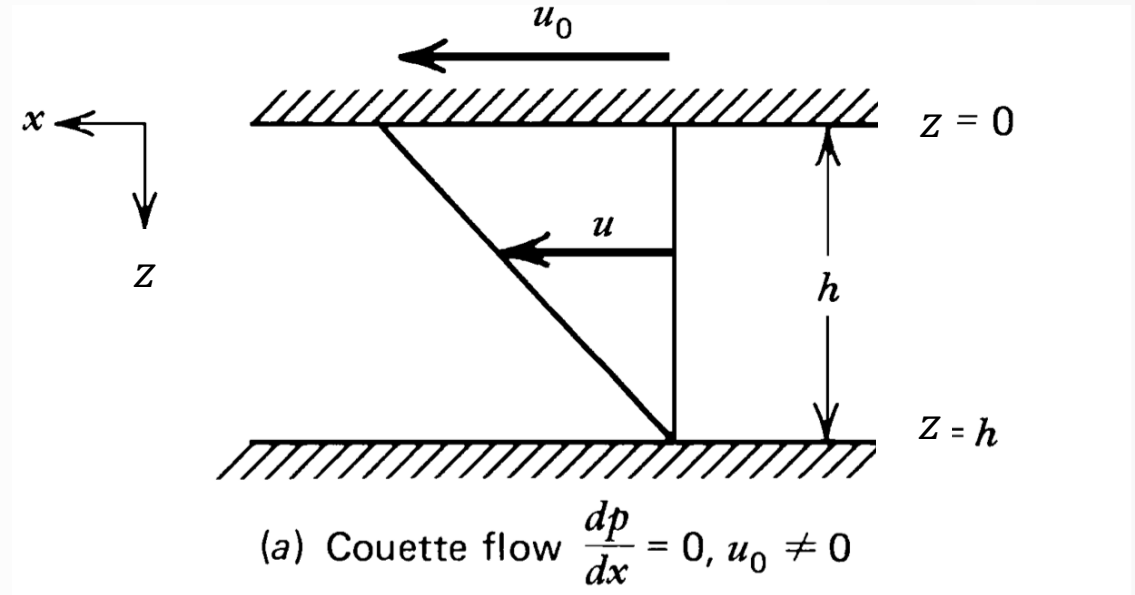


Fig. 6.2a, Turcotte and Schubert, 2014



Styles of linear viscous flow: Poiseuille flow

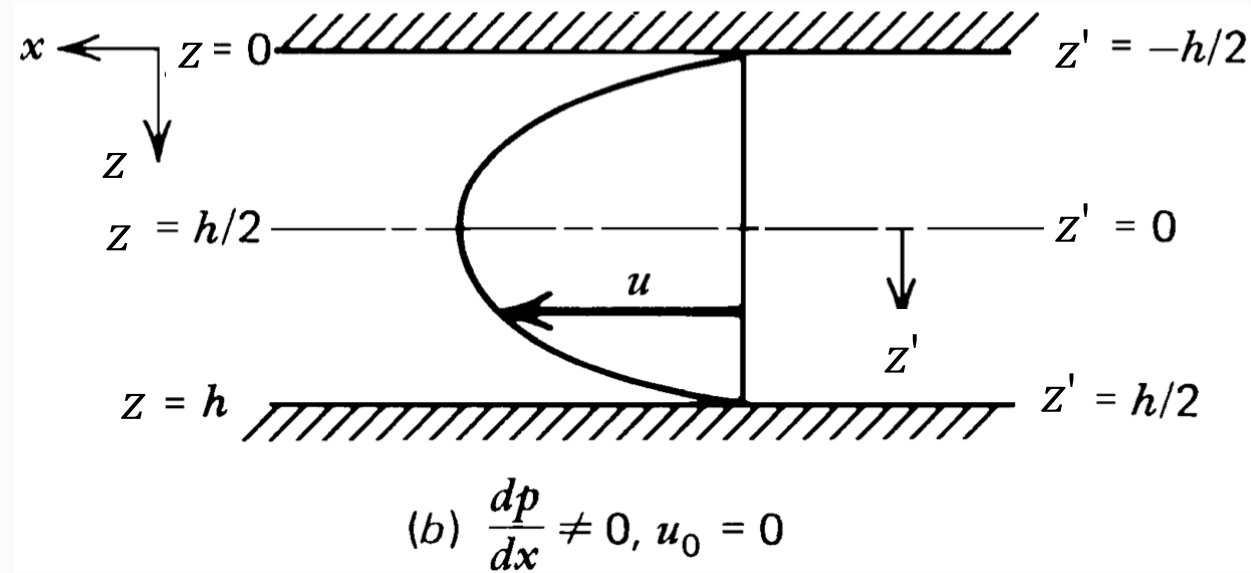


Fig. 6.2b, Turcotte and Schubert, 2002

- **Poiseuille flow** occurs when (1) there is no velocity difference between the walls of the channel and (2) a pressure gradient is applied



Poiseuille flow solution

- Using the same equation as we have previously, we can start with the general solution

$$u = \frac{1}{2\eta} \frac{dp}{dx} (z^2 - hz) - \frac{u_0 z}{h} + u_0$$

- If we set $u_0 = 0$, the velocity solution becomes

$$u = \frac{1}{2\eta} \frac{dp}{dx} (z^2 - hz)$$

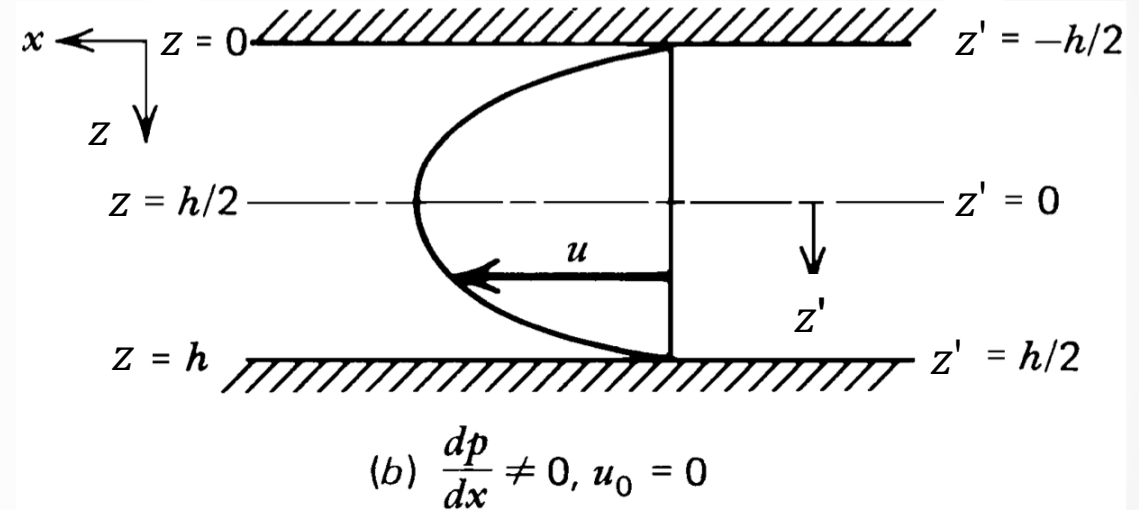


Fig. 6.2b, Turcotte and Schubert, 2002



Salt tectonics



- One example of a geological system that can exhibit both **Couette** and **Poiseuille** flow behavior is the flow of rock salt beneath sedimentary overburden



Temperature dependence of viscosity

- An example from your kitchen:
 - The viscosity of buttercream is temperature-dependent: It is thin and runny when it is warm, and stiff when it is cold.



https://commons.wikimedia.org/wiki/File:Ladybug_cupcakes.jpg



Temperature dependence of viscosity

- In general, rock viscosity depends strongly temperature:

$$\eta = A_0 e^{Q/RT_K}$$

where A_0 and Q are material properties known as the pre-exponent constant and activation energy, R is the universal gas constant and T_K is temperature in Kelvins



Temperature-dependent viscosity (rocks)

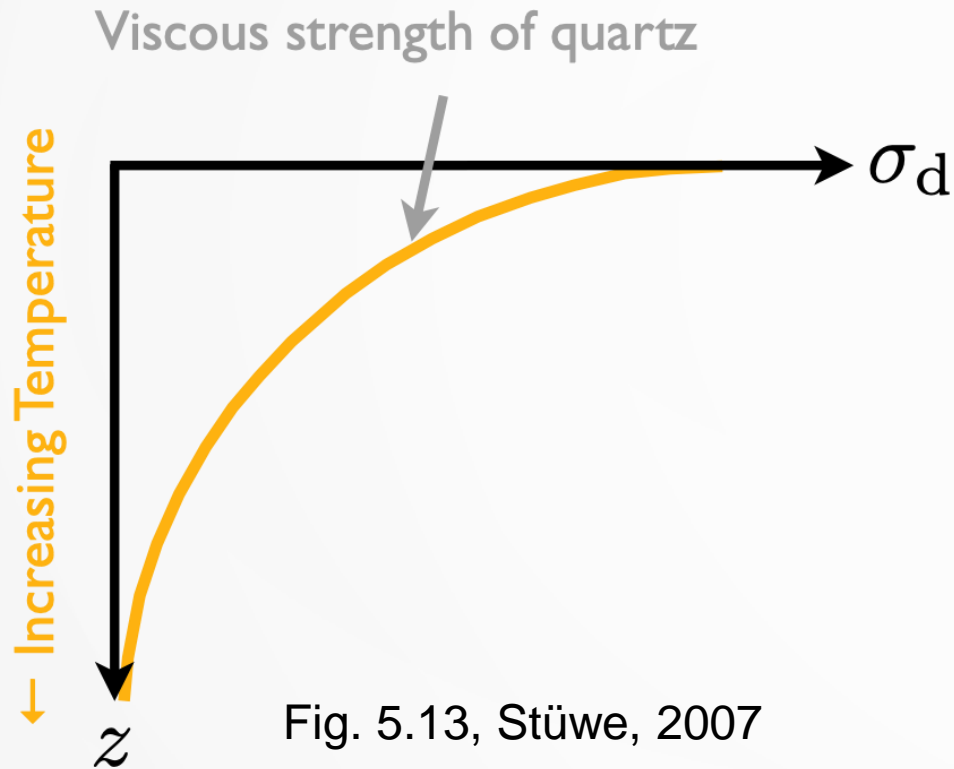


Fig. 5.13, Stüwe, 2007

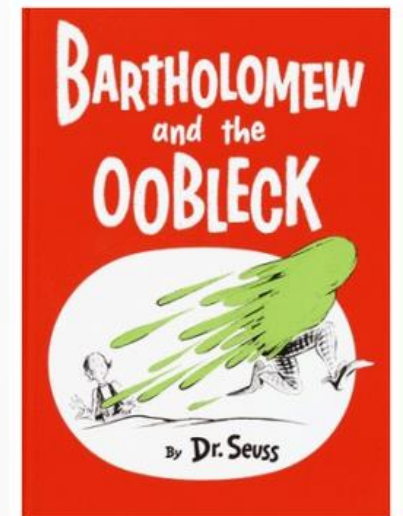
- The viscous strength of quartz rapidly decreases with increasing temperature
- **Viscous strength:** the viscosity η multiplied by a nominal strain rate



Non-linear viscosity



- **Oobleck:**
 - DIY non-linear viscous fluid
 - Suspension of cornstarch in water (2:1)
 - Becomes more viscous when force is applied





Non-linear viscosity

- In general, rocks will deform about 8 times as quickly when the applied force is doubled
 - Relationship between shear stress and strain rate is NOT linear
- Mathematically, we can say

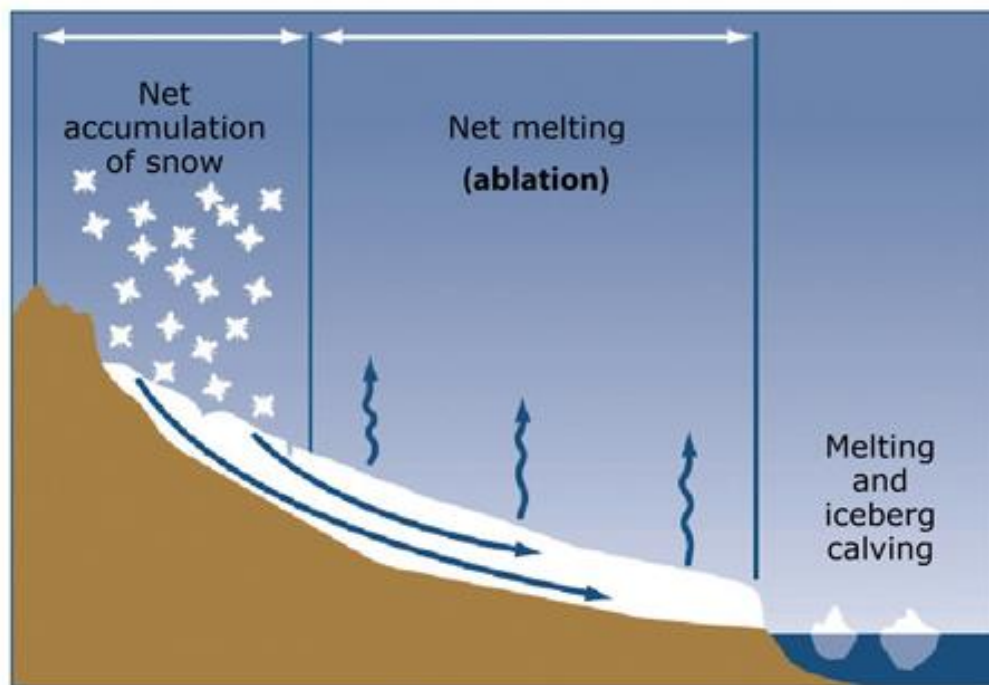
$$\tau^n = A_{\text{eff}} \frac{du}{dz}$$

where n is the **power law exponent** and A_{eff} is a **material constant**

- The power law exponent for many rocks is 2-4
- A_{eff} is similar to η , but has units of $\text{Pa}^n \text{ s}$

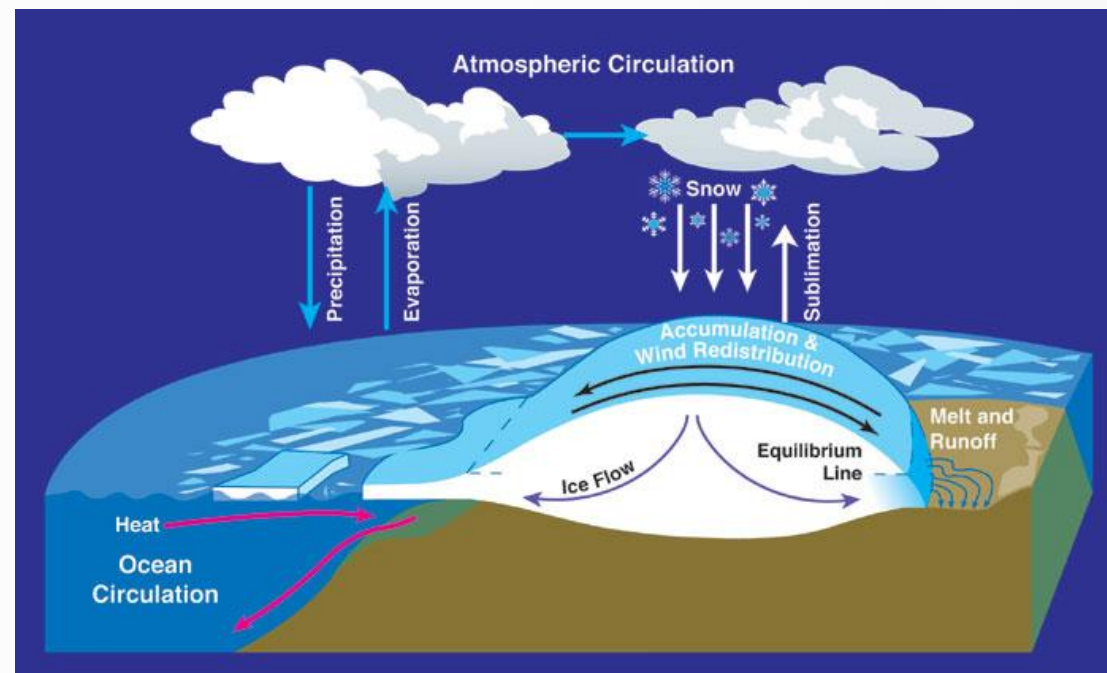


Case study: Flow of ice



From the USGS (<https://pubs.usgs.gov/fs/2009/3046/>)

Glacier mass balance



By NASA. From Wikimedia Commons.

Ice sheet mass balance & atmospheric circulation



Flow of glaciers

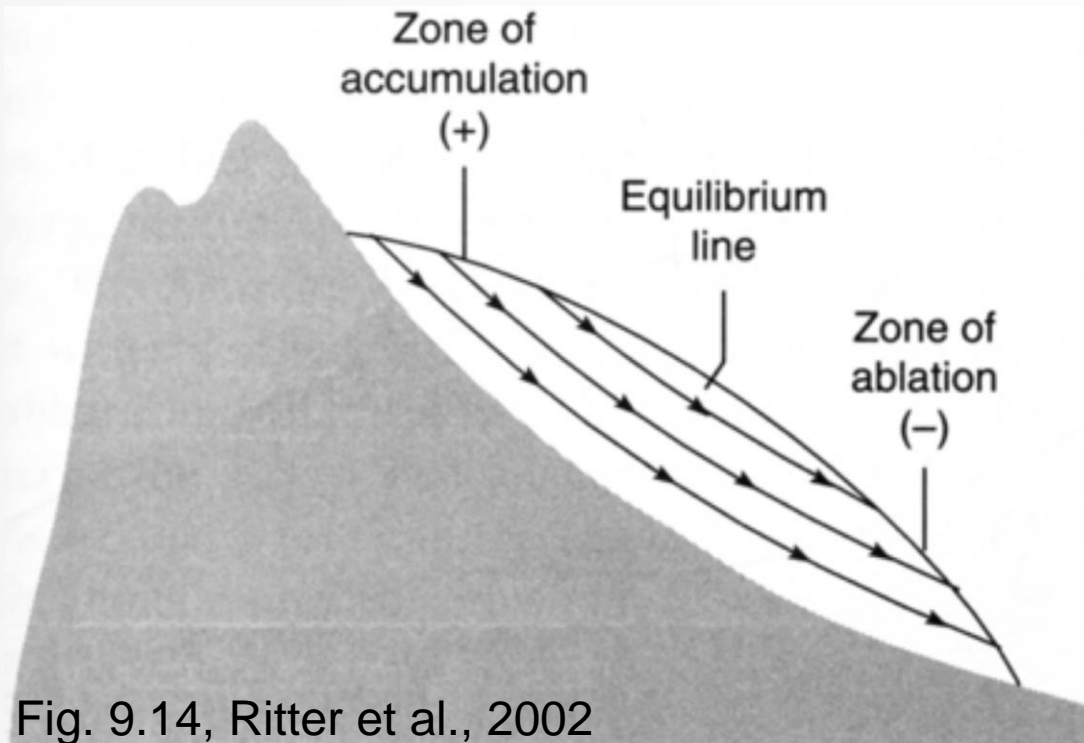


Fig. 9.14, Ritter et al., 2002

- Gravity drives the flow of alpine glaciers from higher elevation zones of **accumulation** to lower elevation zones of **ablation**
- Depending on the temperature of the region and the ice itself, the glacier may either be frozen to the bedrock (**cold-based**) or sliding along the bedrock (**warm-based**)



How do glaciers move?



Briksdal Glacier, Norway

- **Basal sliding**
 - Bottom of the glacier sliding along the substrate
 - Can occur as a result of slip atop a thin water layer, melting/re-freezing or slip atop water-saturated sediment
- **Internal deformation**
 - Ice flow is nonlinear viscous and sensitive to temperature
 - Deformation is concentrated near the bed



How do glaciers move?

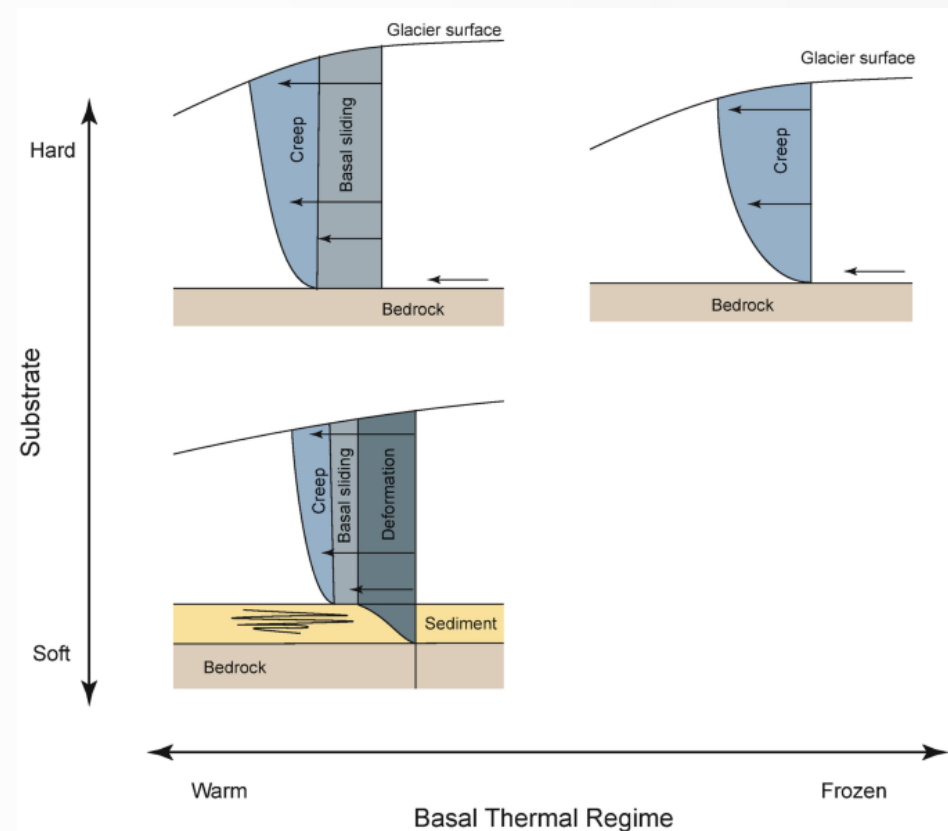


Fig. 35, Jennings & Hambrey (2021)

Bennett (2022)



Recap

- **Viscous flow** is a common deformation behavior for rock and ice, where the deformation rate is proportional to the applied shear stress.
- **Couette** and **Poiseuille** flows refer to end-member behaviors of linear viscous channel flows, and depend on the channel boundary velocities and pressure changes along the channel.
- Most rocks do not exhibit a linear relationship between stress and strain rate (nonlinear viscosity), and their viscosity is strongly temperature-dependent.



Exercise 5: Flow of glaciers

- You will model glacial flow!
- **Due: 3 Dec. at 12:15**

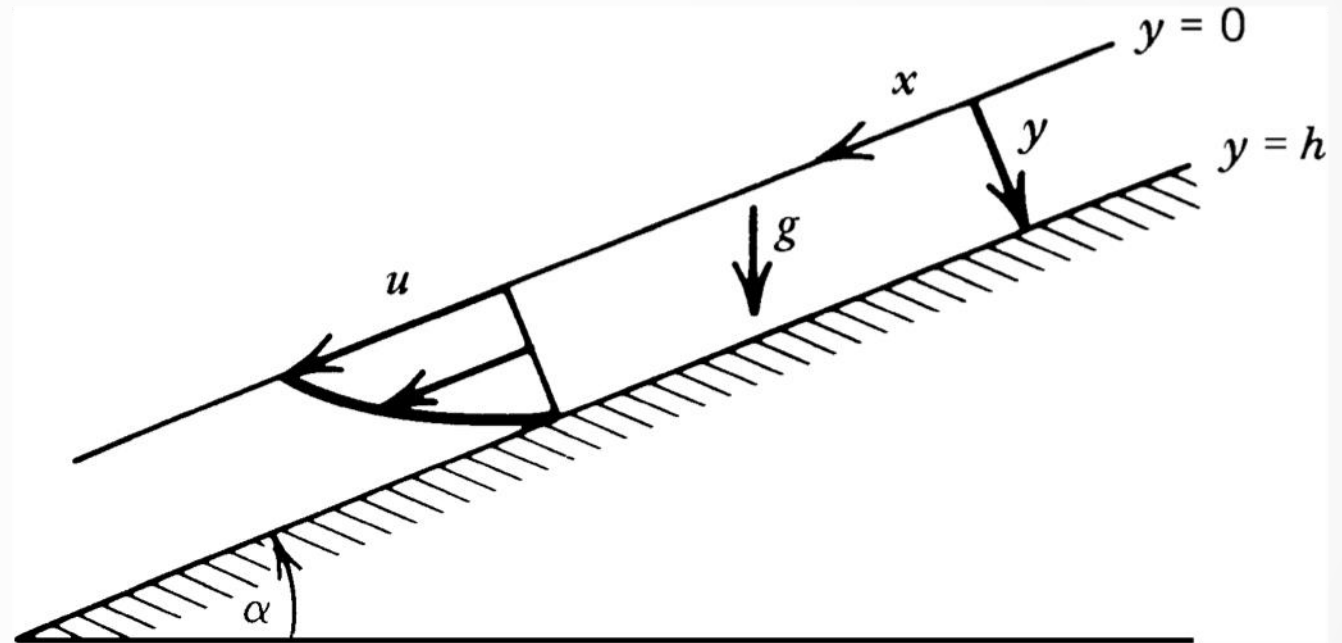


Fig. 6.3, Turcotte and Schubert, 2014



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