Introduction to Quantitative Geology

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Week 4 – Part 1: Geological Advection

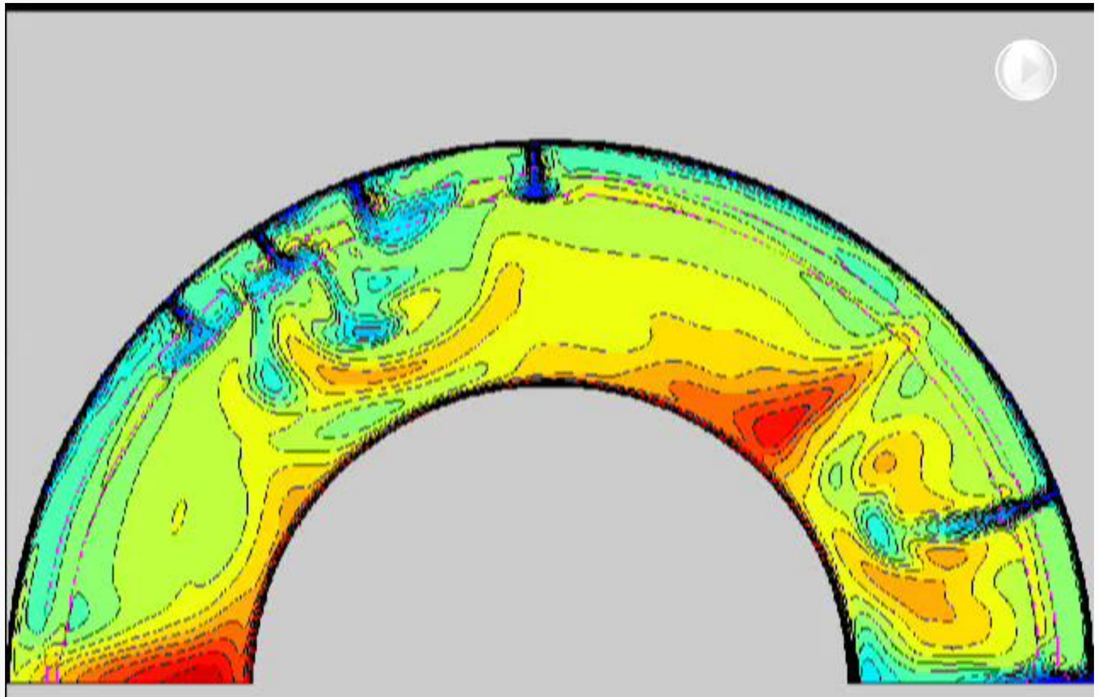
Last week

- **Part 1: Natural diffusion**
  - General concepts of diffusion
  - Mathematical definition
- **Part 2: Basic concepts of thermochronology**
  - What is it?
  - General terms and concepts

This week

- **Part 1: Advection**
  - The advection equation
  - Advection in geological processes
- **Part 2: Erosion, sedimentation, heat transfer**
  - Some definitions
  - What happens with the geothermal gradient?

Definition: Advection



http://homepage.usask.ca/~sab248/

- The lateral translation of some quantity
- In simple terms usually the horizontal movement of a quantity

← Convection of heat: Transfer of heat by physical movement of molecules/atoms within a material and diffusion

Diffusion equation

1) $q = -D\frac{\partial C}{\partial x}$ 2) $\frac{\partial C}{\partial t} = -\frac{\partial q}{\partial x}$

1) Flux proportional to a gradient

2) Conservation: Any change in flux r results in a change in mass/energy

Diffusion

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$

q = flux per unit length
D = diffusivity
C = concentration
x = distance
t = time

Diffusion equation for heat transfer

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

κ = thermal diffusivity
T = temperature
z = distance
t = time

Diffusion of heat

Advection and diffusion equations for heat transfer

Diffusion

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

Advection

$$\frac{\partial T}{\partial t} = v_z \frac{\partial T}{\partial z}$$

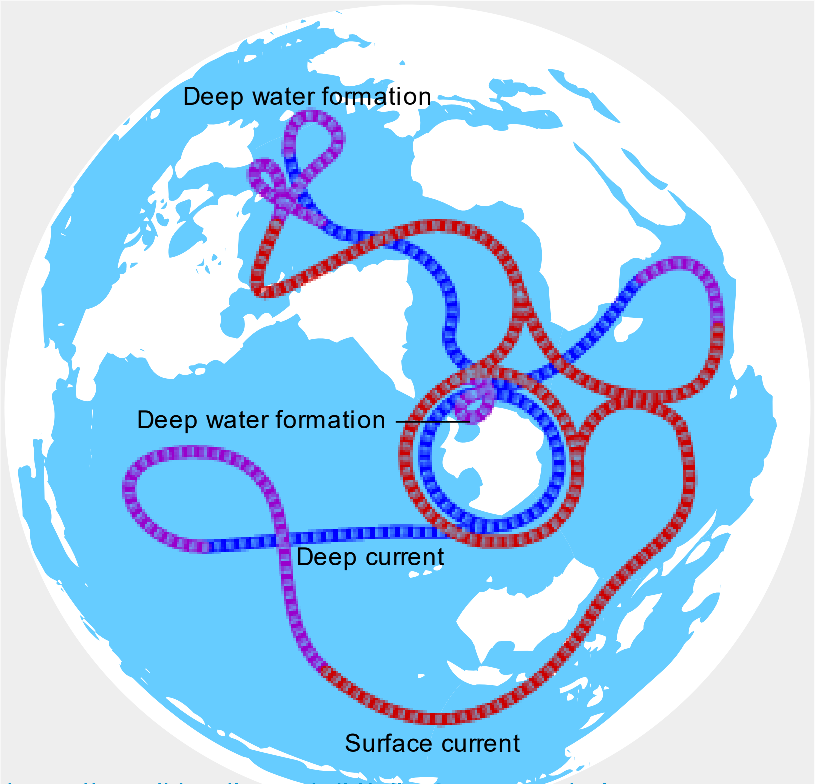
v_z: Advection coefficient

- **Diffusion:** Change in mass/energy with time proportional to the curvature (change in gradient)
- **Advection:** Change in mass/energy with time proportional to the gradient

Advection (+ diffusion) as a geological process

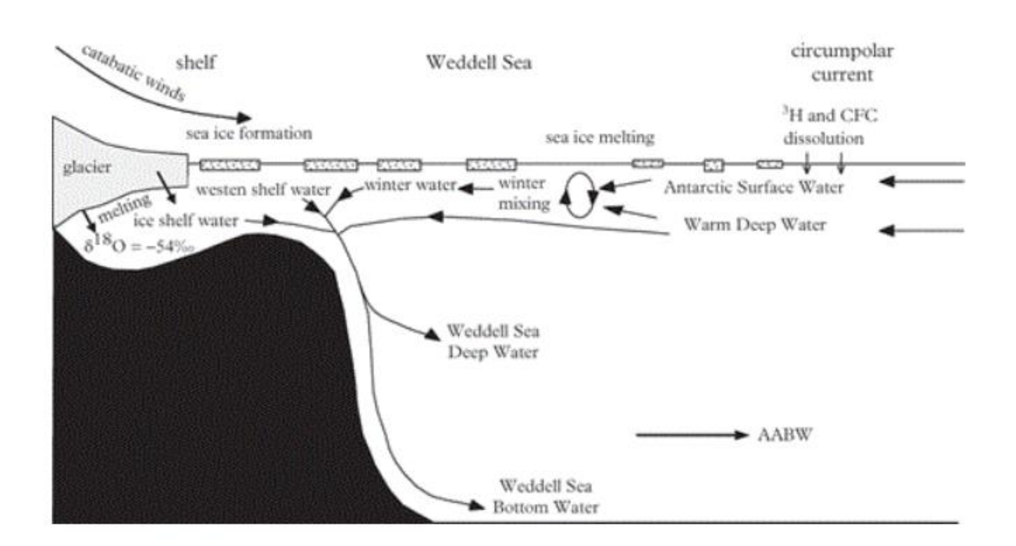
- **Examples/ case studies**
  - Case study 1: Thermohaline circulation
  - Case study 2: Submarine groundwater discharge
  - Case study 3: Rivers

Oceans: The thermohaline circulation



https://en.wikipedia.org/wiki/File:Conveyor_belt.svg

The global conveyor belt

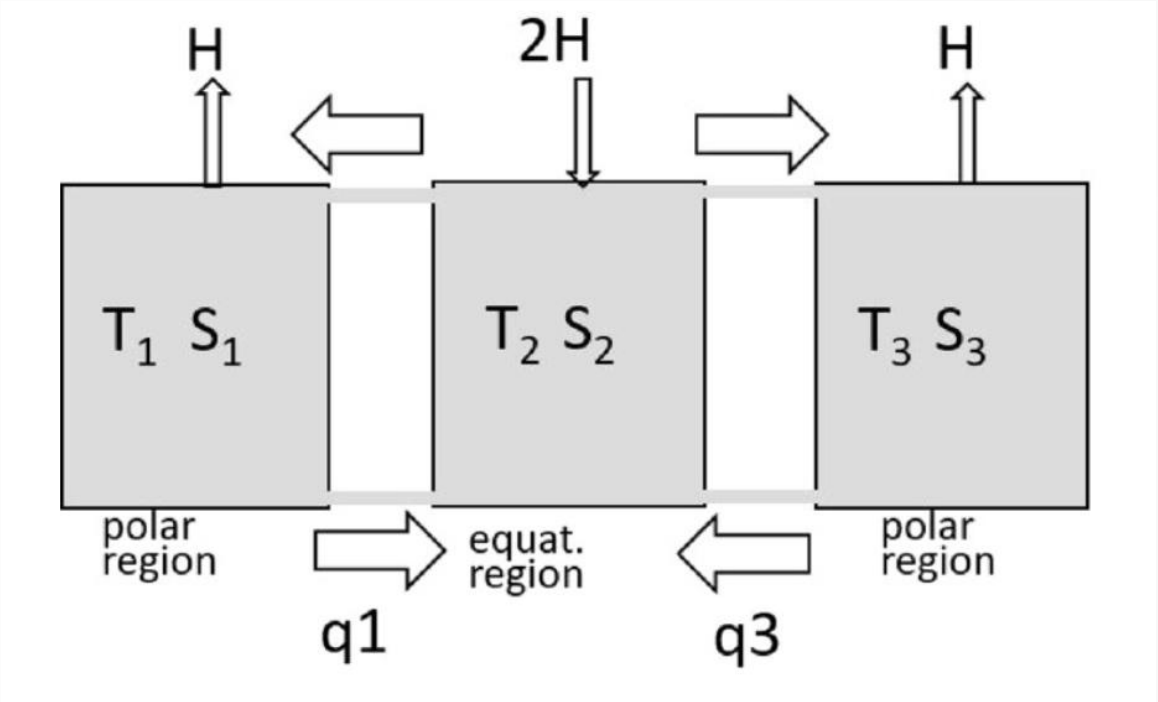


Deep water formation in the Weddell Sea

Roy-Barmnan & Jaendal (2016)

Oceans: The thermohaline circulation

- Models and simulations for
  - Analysing driving forces and perturbations
  - Finding equilibrium conditions and tipping points
  - Testing sensitivity to changes in
  - density, salinity, temperature



Box model for thermohaline circulation after Wehlander 1986, (Neitzel & Gehrig 2022)

Fun fact: ”Friendly Floatees”
Advection of rubber ducks in the ocean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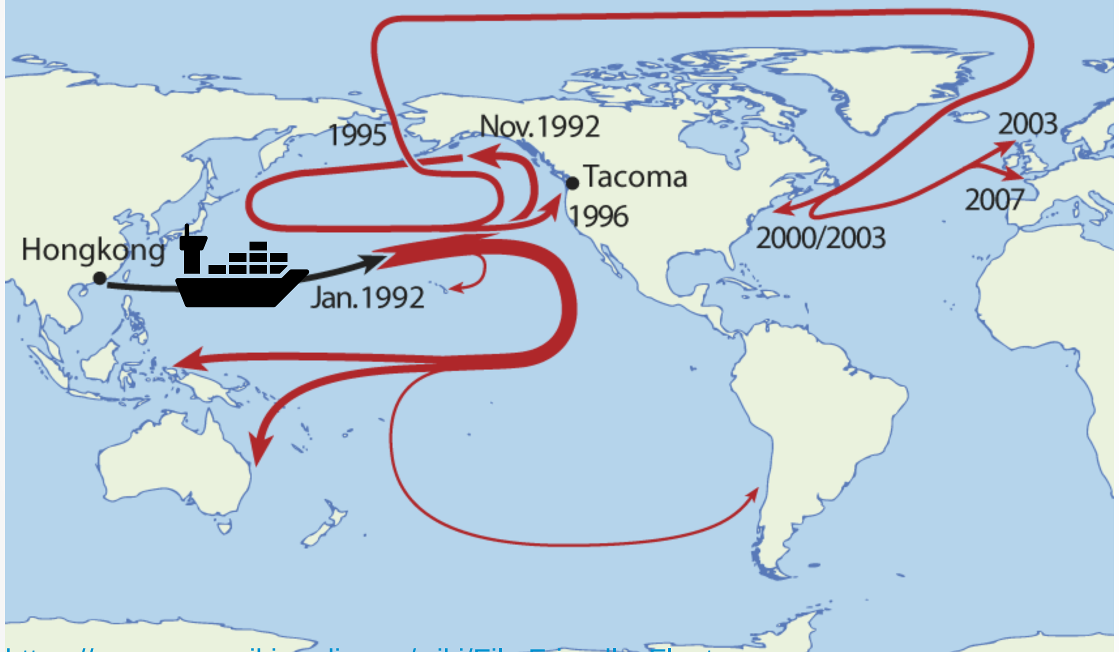


https://commons.wikimedia.org/wiki/File:Friendly_Floatees.png

Journey of the ”Friendly Floatees”

- 28,800 yellow ducks, red beavers, blue turtles and green frogs went overboard a cargo ship in 1992
- The spilled bath toys washed ashore were used to study and model ocean currents

[1] Rebecca, Kamden and Kaitlyn Johnson, Science Museum Group Collection

Submarine groundwater discharge

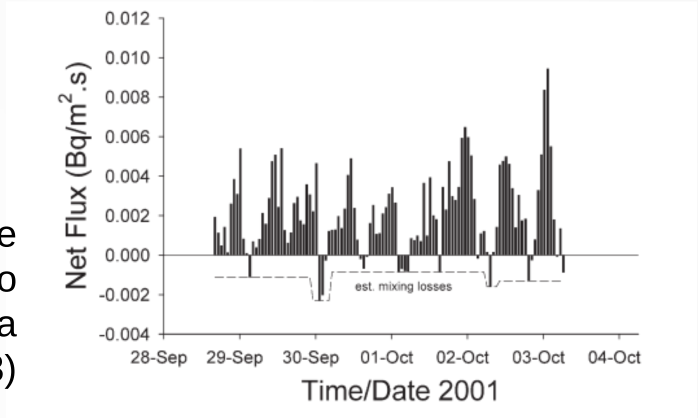
- Groundwater directly discharges into the sea

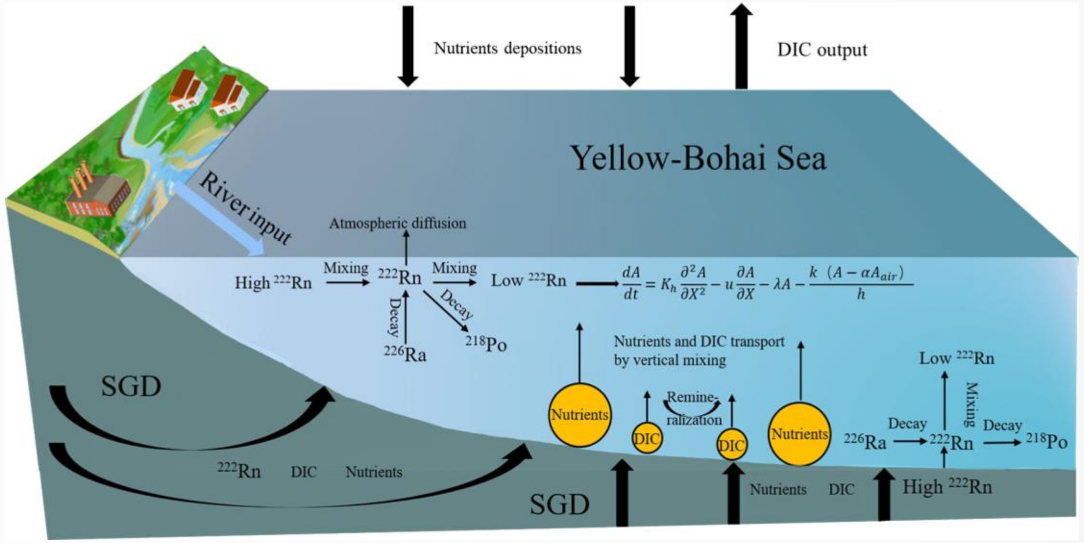
➢ Potential pollution of coastal waters

- How to measure flux of groundwater to coastal waters?

➢ Tracer: ^{222}Rn ($t_{1/2}$=3.83 days, ~1000x enriched in groundwater)

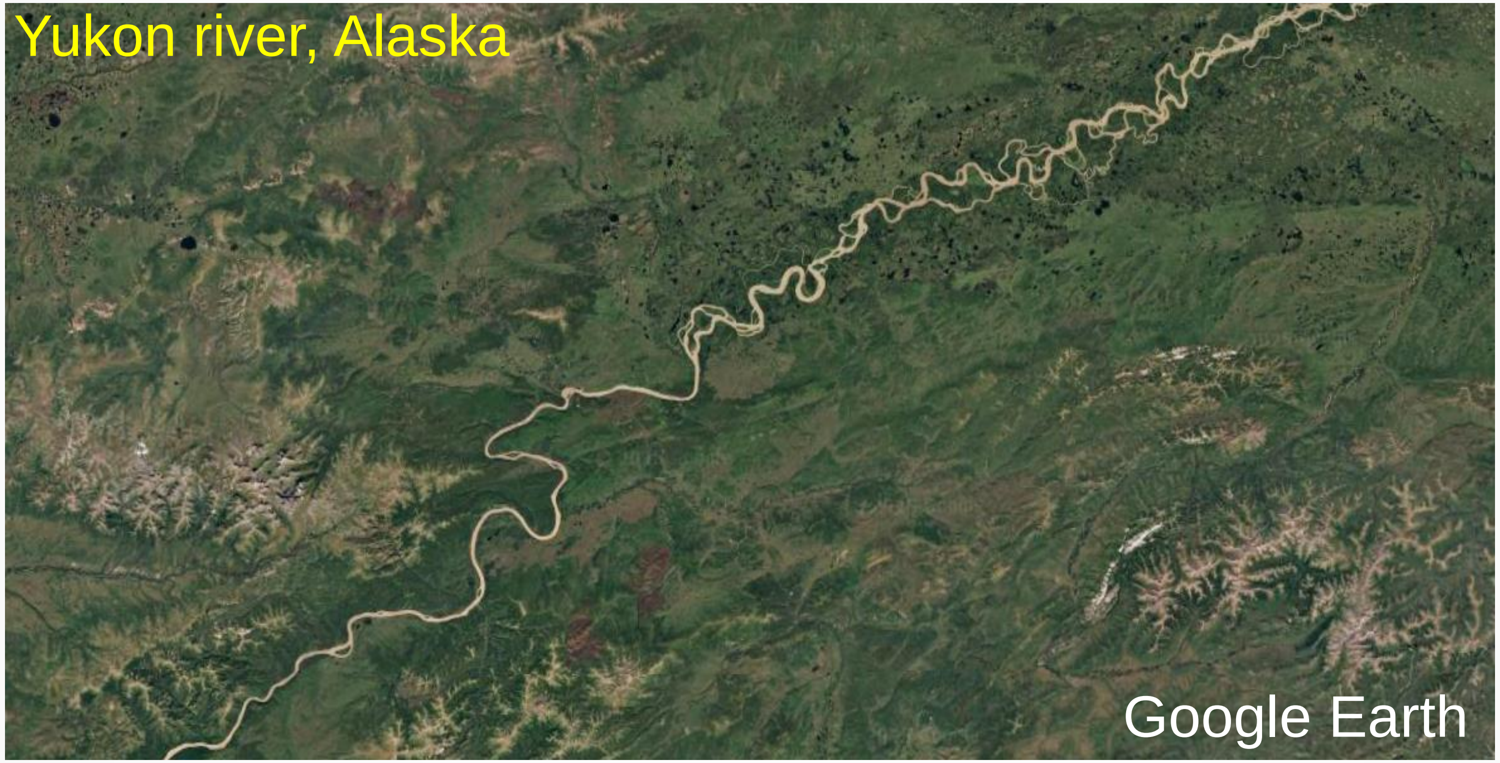
Net radon flux in the Gulf of Mexico Burnett & Dulaiova (2003)





Model of radon flux in coastal waters Wang et al. (2021)

River channels



River channels

- **Bedrock channel**
  - Bed and banks made of bedrock
  - Ability of a river to remove sediment is greater than sediment supply
  - **supply limited**

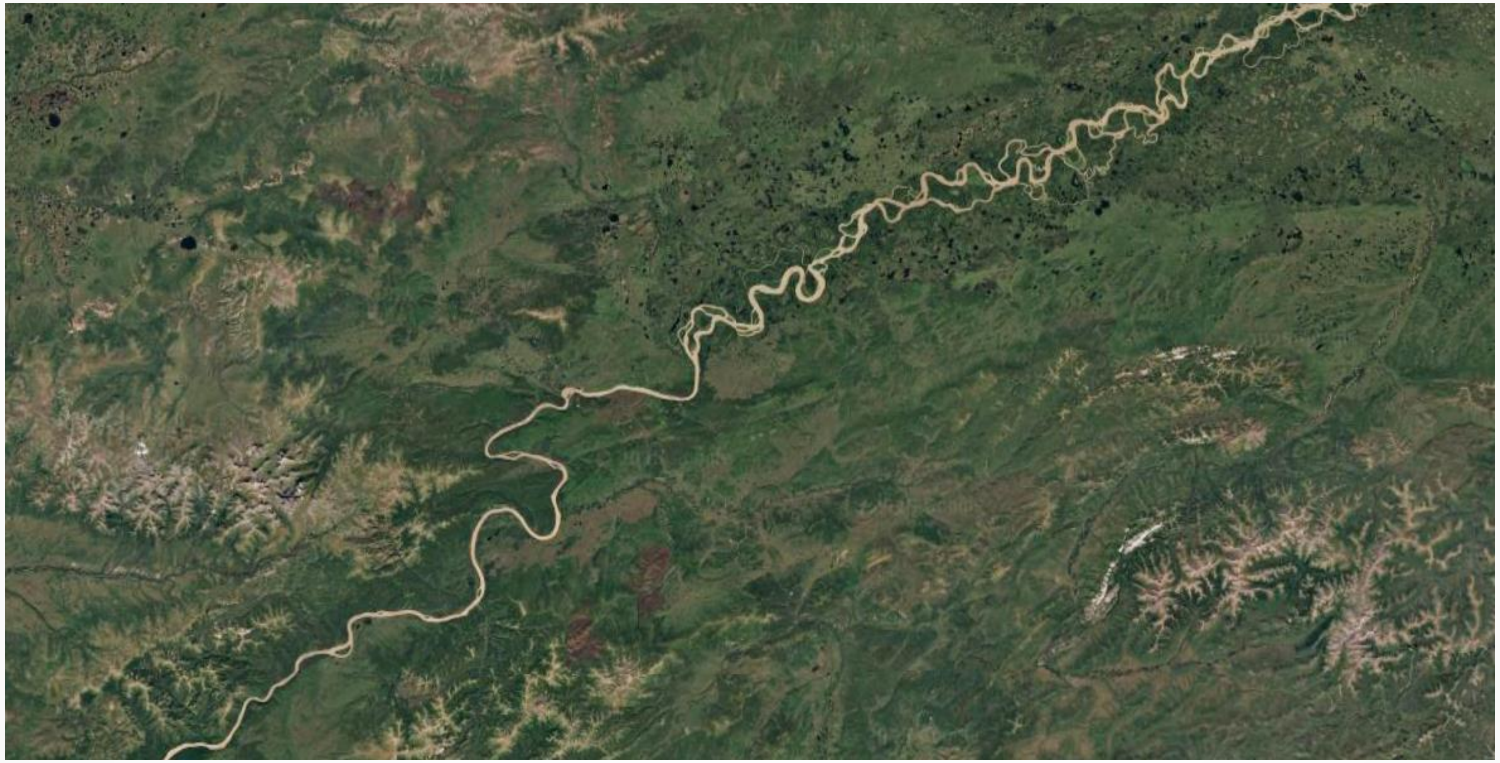
- **Alluvial channel**
  - Bed and banks made up of mobile (unconsolidated) sediment
  - Sediment supply is greater than (or equal to) the amount of sediments that the river moves downstream.
  - ***transport limited***

River channels: which is which?

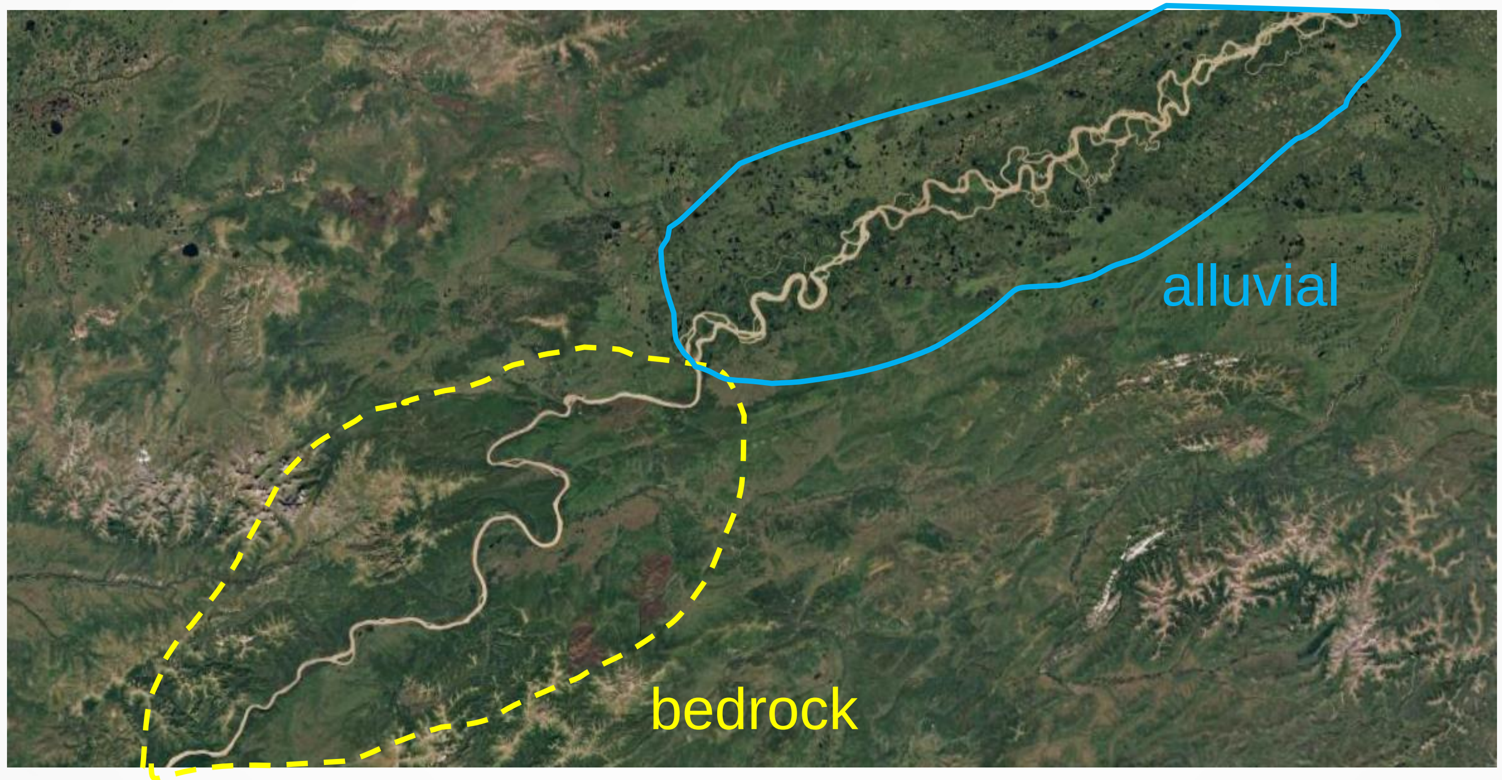
https://commons.wikimedia.org/wiki/File:Hornbach_bild_vom_2_8_177.jpg

Gomez-Heras et al. (2019)

Yukon in Alaska, bedrock or alluvial?

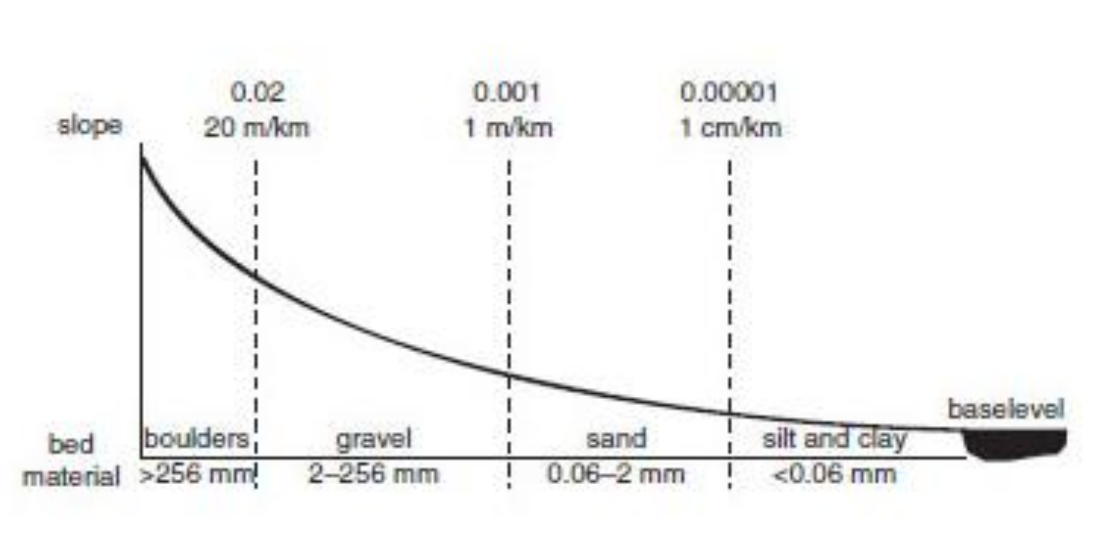


Yukon in Alaska, bedrock or alluvial?



What does that have to do with advection?

- **River profiles**
  - Longitudinal cross-section from source (headwaters) to mouth (baselevel)
  - Shows changes in river slope (gradient)
  - Shape of the river profile is influenced by diffusive and advective processes



Anderson & Anderson (2010)

What does that have to do with advection?

- Bedrock channel
  - advection dominates

- Alluvial channel
  - diffusion dominates

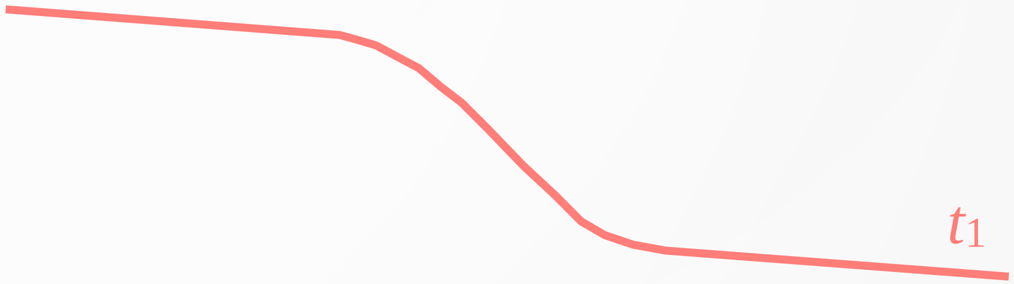


Fig. 1.7, Pelletier, 2008

What does that have to do with advection?

- Bedrock channel

$$\frac{\partial h}{\partial t} = c\frac{\partial h}{\partial x}$$

- Alluvial channel

$$\frac{\partial h}{\partial t} = -\kappa\frac{\partial^2 h}{\partial x^2}$$

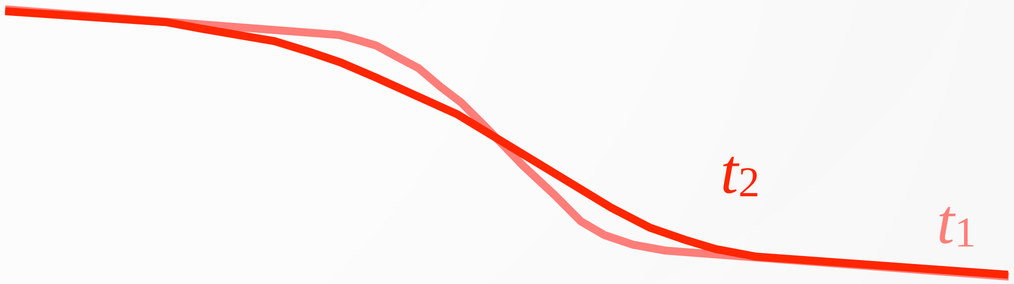
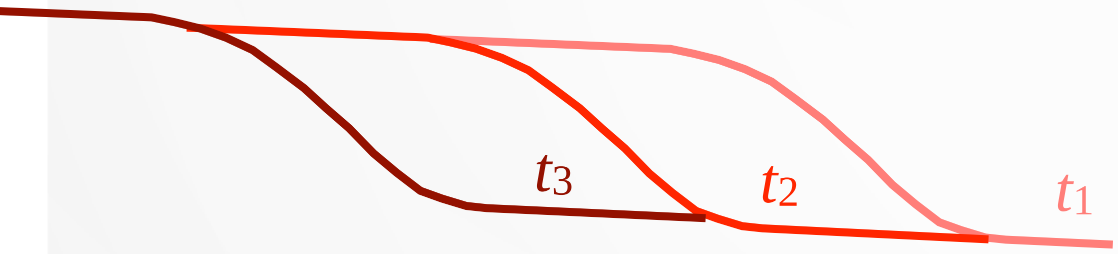


Fig. 1.7, Pelletier, 2008

What does that have to do with advection?

- Bedrock channel
  - Rate of erosion is directly proportional to hillslope gradient



- Alluvial channel
  - Rate of erosion depends on change in hillslope gradient (curvature)

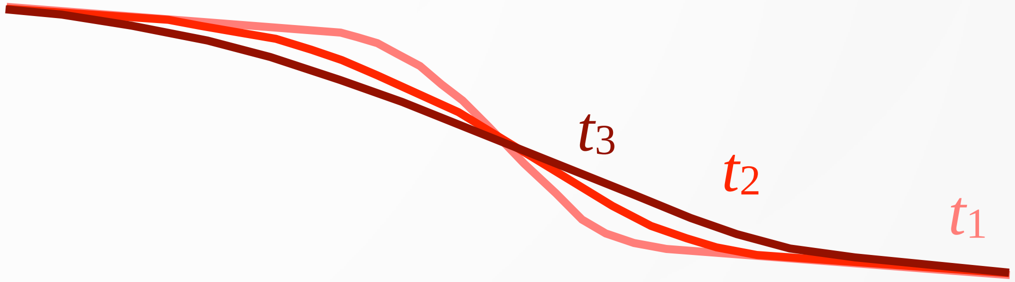
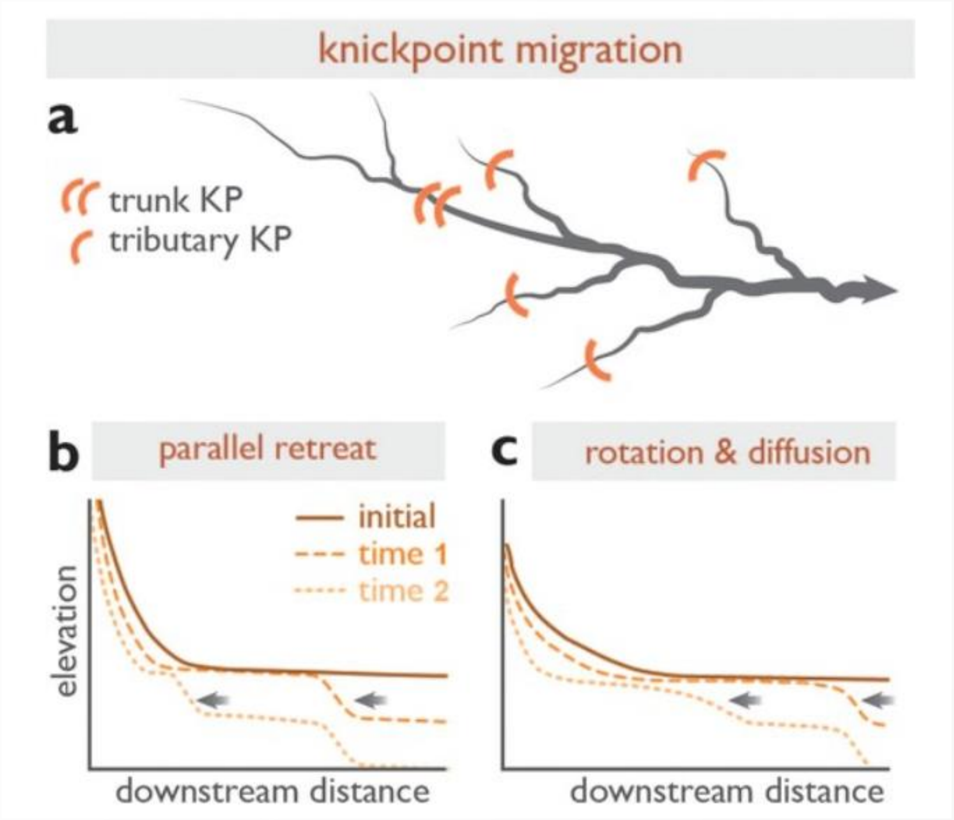


Fig. 1.7, Pelletier, 2008

River channel profiles: knickpoints

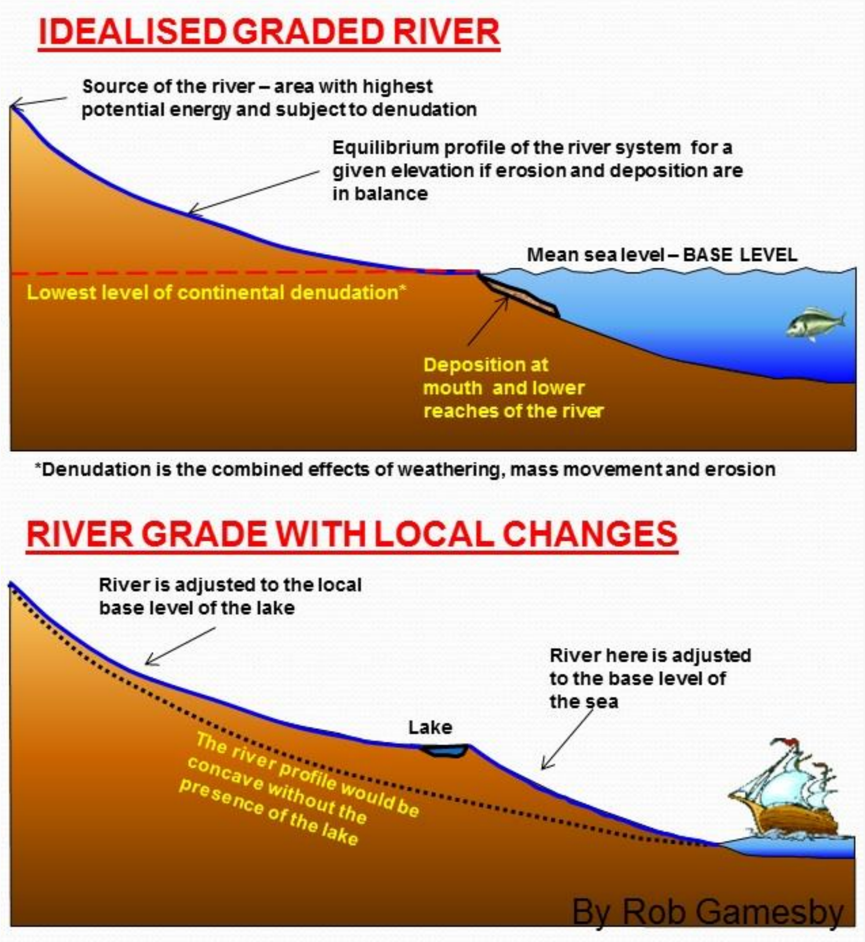
- **Knickpoint:**
  - Sharp change in river slope
  - Caused by perturbations to the river system
  - Migrates upstream due to erosion



Jonell et al. (2023)

River channel profiles: knickpoints

- Knickpoints can tell us something about
  - **Tectonics:** baselevel drop, uplift of the upstream drainage area
  - **Lithology:** differences in rock erodibility
  - **Climate:** Changes in precipitation



https://www.coolgeography.co.uk/A-level/AQA/Year%2012/Rivers_Floods/Rejuvenation/Rejuvenation.htm

Which process ”wins”? Advection or diffusion?

- **The Peclet number *Pe*:**

$$\boldsymbol{Pe = \frac{v_z\, L}{k}}$$

where ***k*** is the diffusivity, $\boldsymbol{v_z}$ is the advection coefficient, **L** is the characteristic length

- Pe>0: advection dominates
- Pe<0: diffusion dominates

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