



Introduction to Quantitative Geology

Lecturer: Ann-Kathrin Maier

Week 3 – Part 2: Basic concepts of thermochronology



This week

- **Part 1: Natural diffusion**
 - General concepts of diffusion
 - Mathematical definition
- **Part 2: Basic concepts of thermochronology**
 - What is it?
 - General terms and concepts

Campo Imperatore, Apennines,
Italy



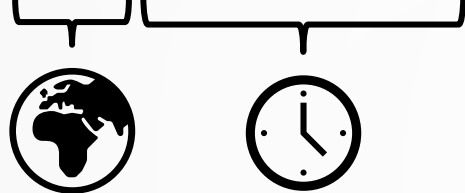
Why thermochronology?

Popular dating technique for studying long-term tectonic and erosional processes (typically over millions of years)



Geochronology vs thermochronology

Geochronology

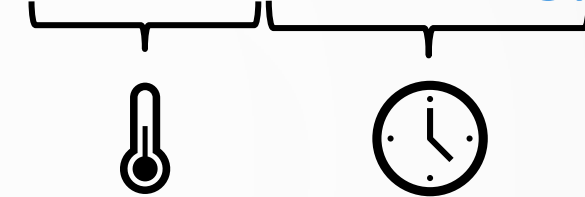


The science of **dating** geological materials.

Geochronological ages

Generally interpreted as ages of the materials (**crystallization ages**).

Thermochronology



The science of inferring **thermal histories** of minerals and rocks.

Thermochronological ages

Interpreted as the time since the material cooled below a given temperature (**cooling ages**)



Thermochronology applications

- Paleotopography reconstruction

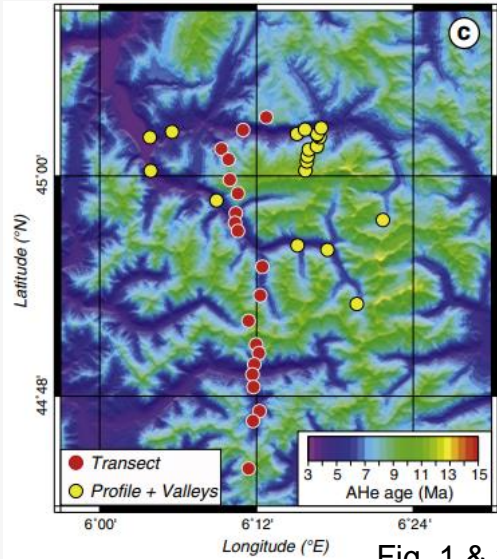
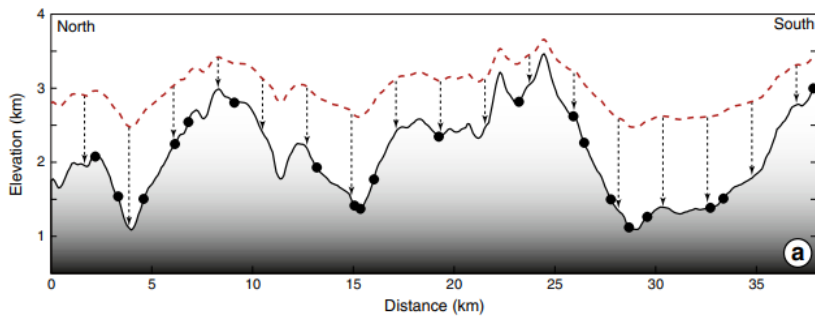


Fig. 1 & 2, Valla et al. (2010)



- Weathering and paleoclimate changes

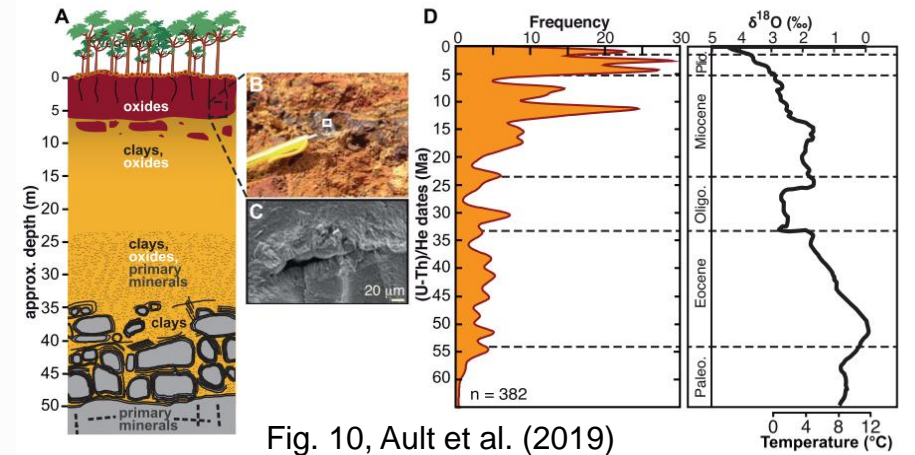


Fig. 10, Ault et al. (2019)

- Exhumation, erosion, deposition, burial

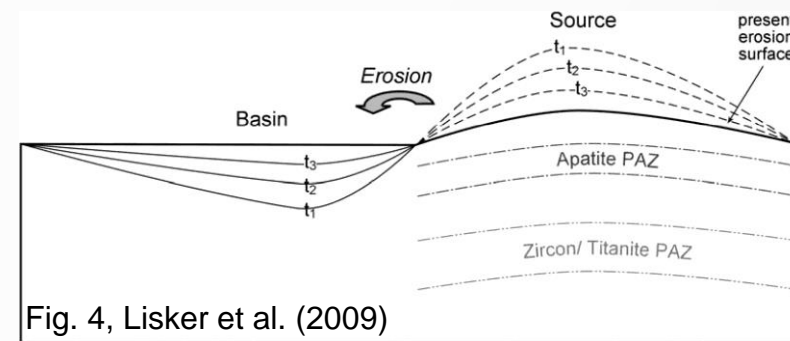


Fig. 4, Lisker et al. (2009)

And more!



What is a thermochronometer?

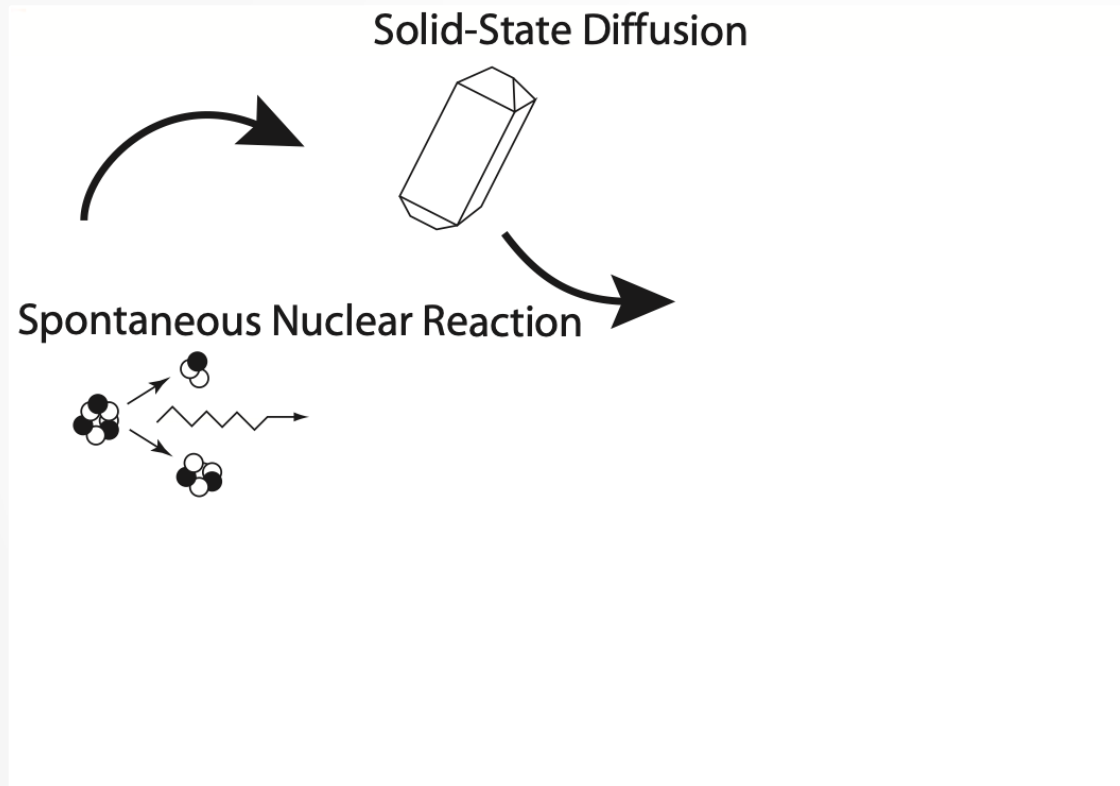


Fig 1.1, Braun et al., 2006

- **Thermochronometer**

A radio-isotopic system consisting of:

- a radioactive parent
- a radiogenic daughter isotope or crystallographic feature
- the mineral in which they are found



Thermochronometry & Thermochronology

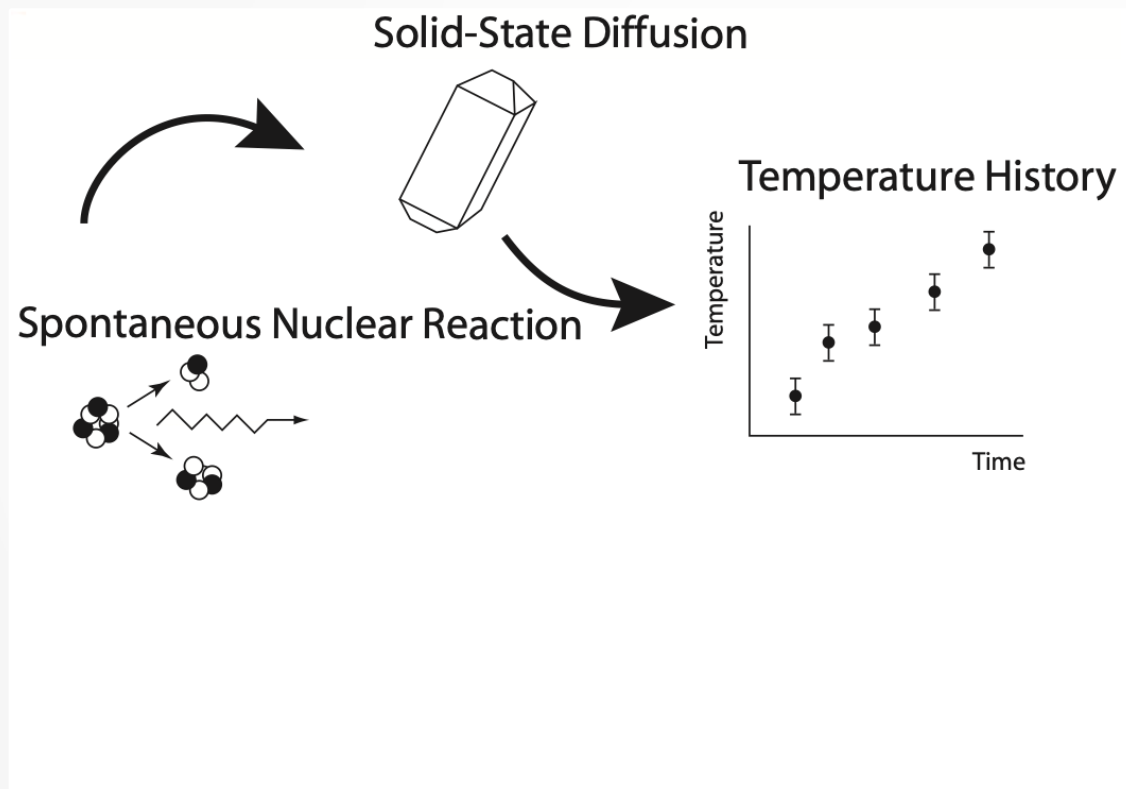


Fig 1.1, Braun et al., 2006

- **Thermochronometry**
The analysis, practice, or application of a thermochronometer to understand thermal histories of rocks or minerals.
- **Thermochronology**
The thermal history of a rock, mineral, or geologic terrane.



What can you do with thermochronology?

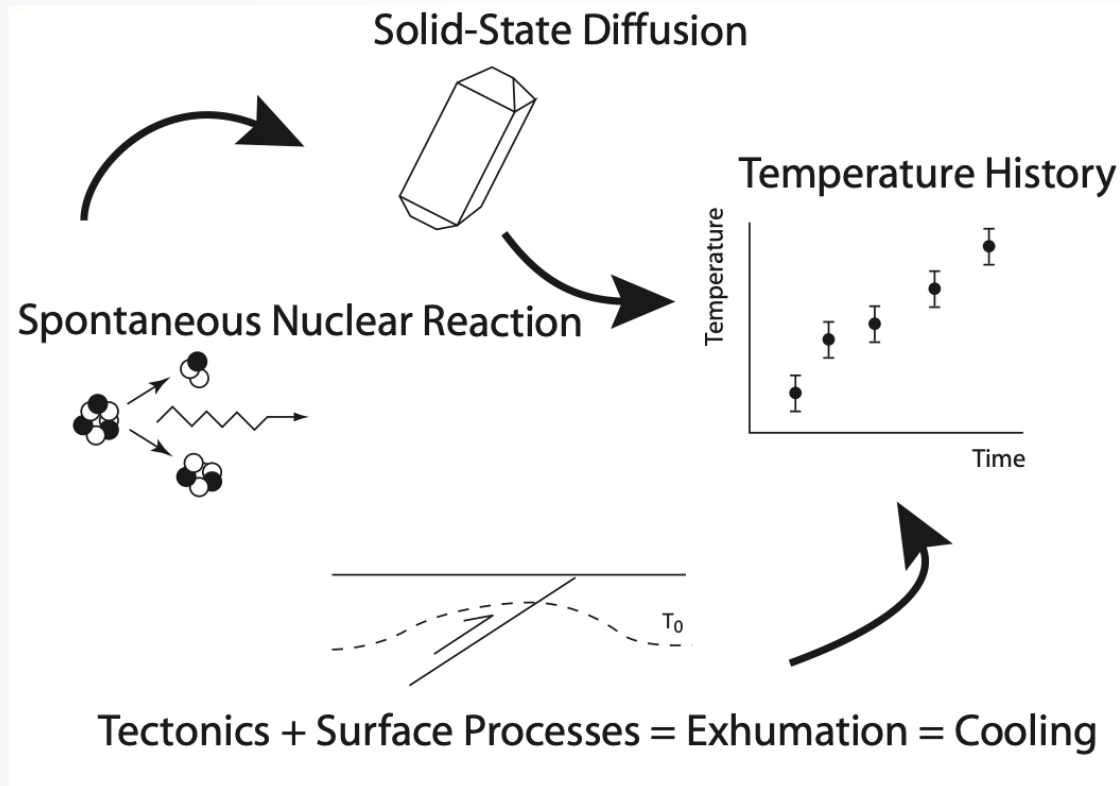
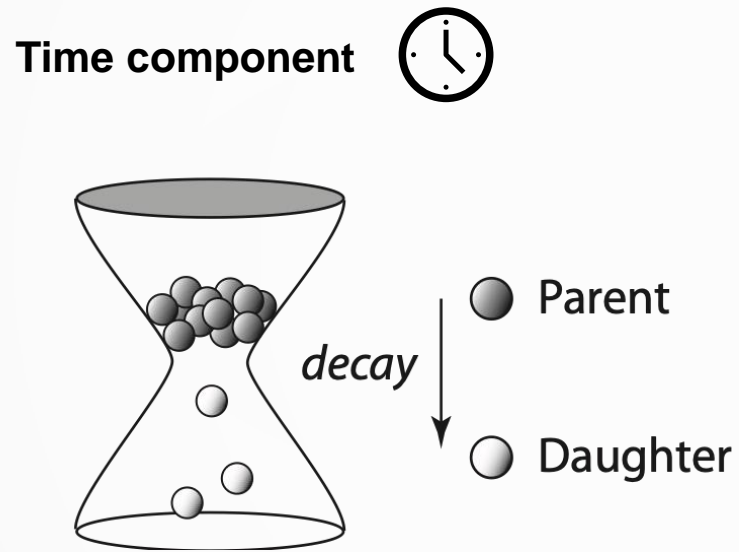


Fig 1.1, Braun et al., 2006

- Goal of most modern applications:
Use the recorded thermal history to provide insight into past tectonic or erosional (surface) processes.
- Link the temperature to which a thermochronometer is sensitive to a depth in the Earth



How does it work?



- **Daughter products** are continually produced within a mineral as a result of radioactive decay

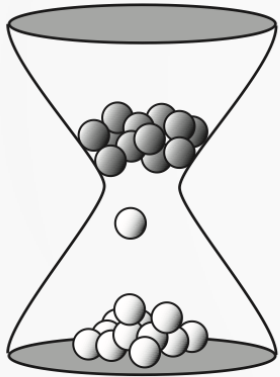
Fig 1.3, Braun et al., 2006



How does it work?

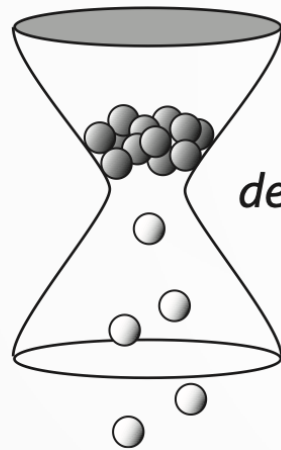
Temperature component 

Closed System



Low T

Open System



High T

decay

● Parent

● Daughter

Thermally activated diffusion:

- The temperature below which the daughter products are retained depends on the daughter product and the host mineral.

Fig 1.3, Braun et al., 2006



Closure temperature and partial retention zone

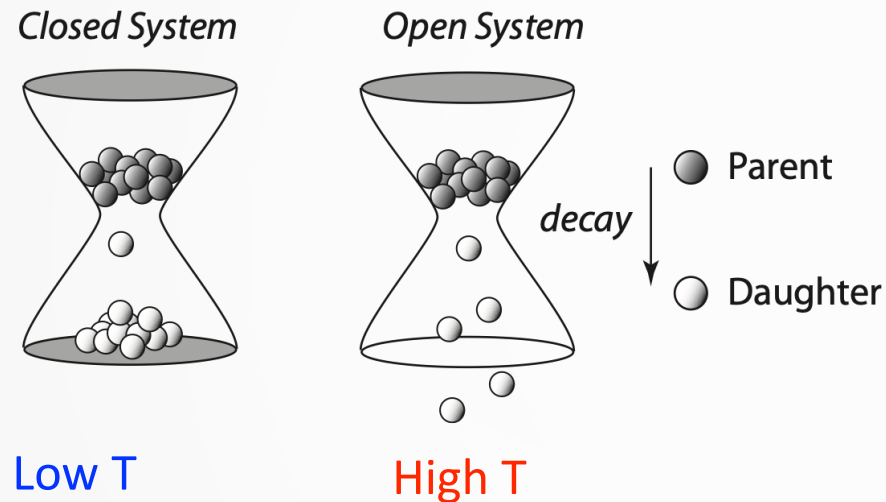
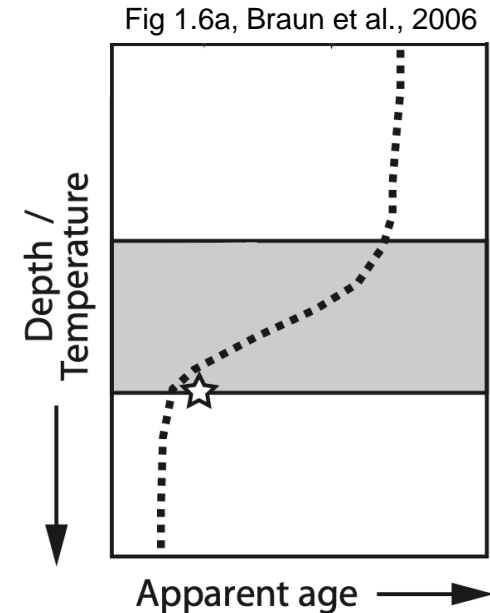


Fig 1.3, Braun et al., 2006



The transition from an open to a closed system is not instantaneous at a given temperature. It occurs over a temperature range known as the **partial retention** (or **partial annealing**) **zone**.



Closure temperature and partial retention zone

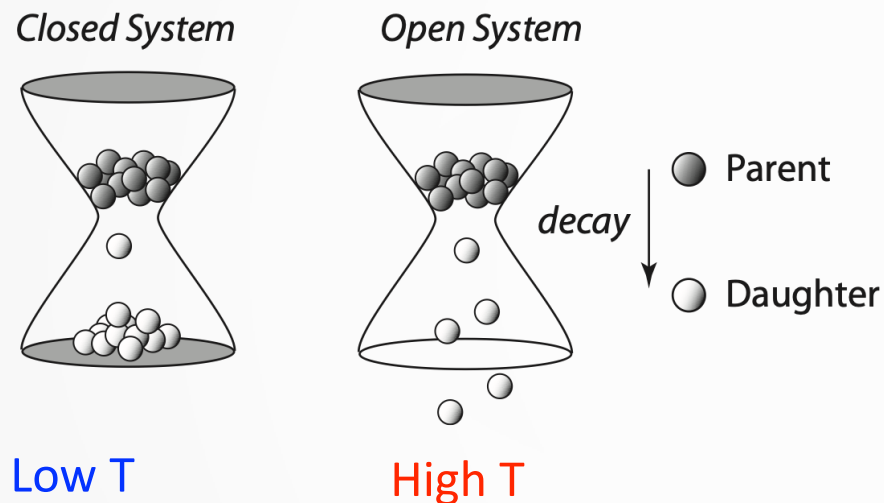
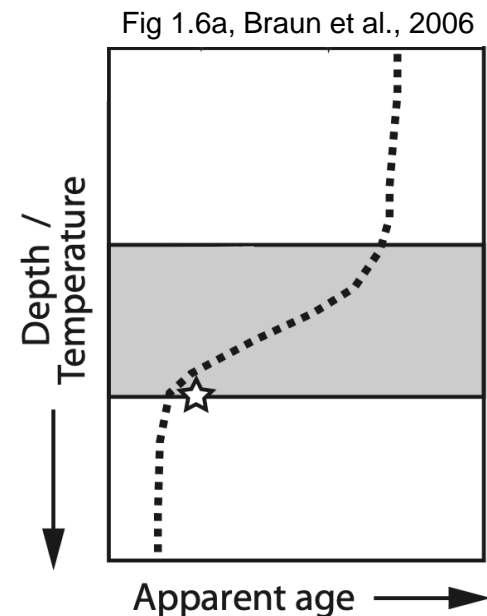


Fig 1.3, Braun et al., 2006



The **partial retention zone** temperature range spans from the point at which nearly all produced daughter products are lost to diffusion to where they are nearly all retained



Effective closure temperature

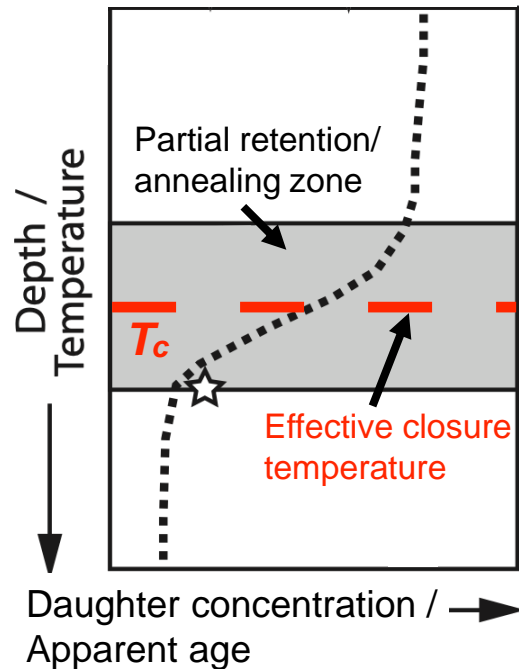


Fig 1.6a, Braun et al., 2006

- Defined by Dodson (1973):

The **closure temperature** is the 'temperature of a thermochronological system at the time corresponding to its apparent age'

*when cooling is monotonic (no reheating)



Effective closure temperature

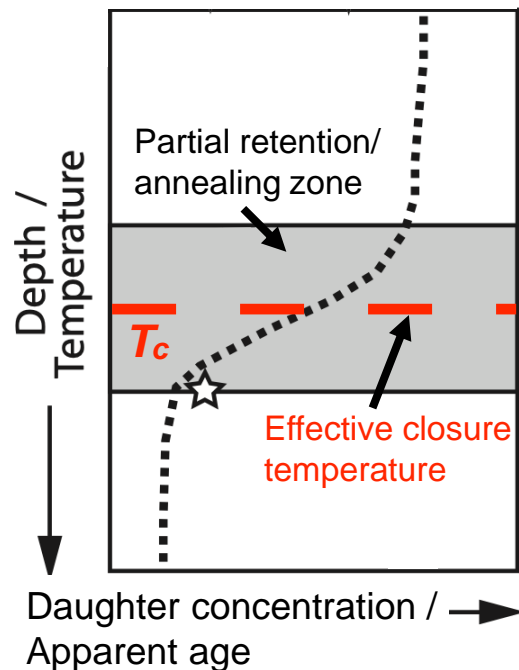


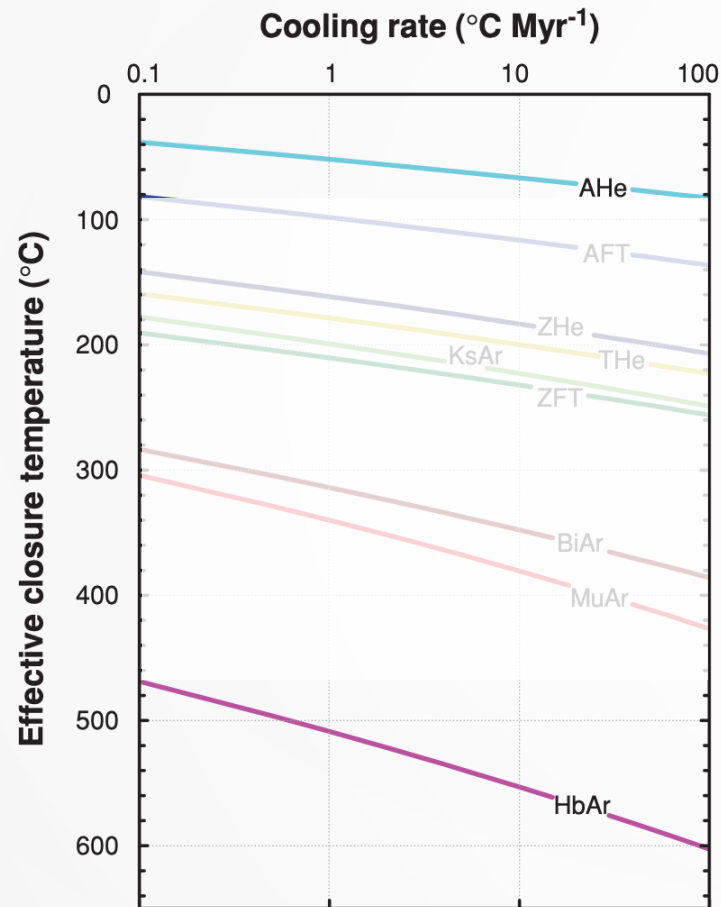
Fig 1.6a, Braun et al., 2006

- Allows us to relate a measured age to a temperature (and depth) in the Earth.
- **Caution!** Closure temperatures vary as a function of the thermochronological system, diffusion domain (mineral) size, chemical composition and cooling rate.



Effective closure temperature

Influence of cooling rate



Reiners and Brandon, 2006

- In general:
 - Effective closure temperature for a given thermochronometer system increases with increasing cooling rate.
 - Absolute difference in effective closure temperature is larger for higher temperature thermochronometers
 - ~40°C for ⁴He in apatite
 - ~130°C for ⁴⁰Ar in hornblende



Radioisotopic chronometer ages

- The general equation for an isotopic age is

$$t = \frac{1}{\lambda} \ln \left(1 + \frac{N_d}{N_p} \right)$$

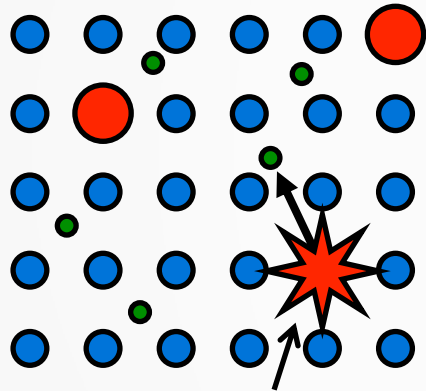
where t is the isotopic age, λ is the radioactive decay constant, N_d is the concentration of the daughter product and N_p is the concentration of the parent isotope

- **For thermochronometers:** Concentration of the daughter product will vary because of **radioactive decay, AND solid-state diffusion.**






Solid-state diffusion

Parent and daughter isotopes in a crystal



Alpha decay

-  **Parent isotope**
-  **“Normal” atom**
-  **Daughter isotope**

- Thermochronometer daughter products are mobile and diffuse within the crystal
- Their diffusion can be modelled using the standard **diffusion equation**

$$\frac{\partial N_d}{\partial t} = D(T) \frac{\partial^2 N_d}{\partial x^2} + P \quad \mathbf{1D}$$

where $D(T)$ is the temperature dependent diffusivity (see next slide), $\partial^2 N_d / \partial x^2$ is the second derivative of the daughter product concentration and P is the daughter production rate (often assumed to be constant over the age of a sample)



Temperature-dependent diffusion

- **Temperature dependence** for diffusion is typically modelled as

$$\frac{D(T)}{a^2} = \frac{D_0}{a^2} e^{-E_a/(RT_K)}$$

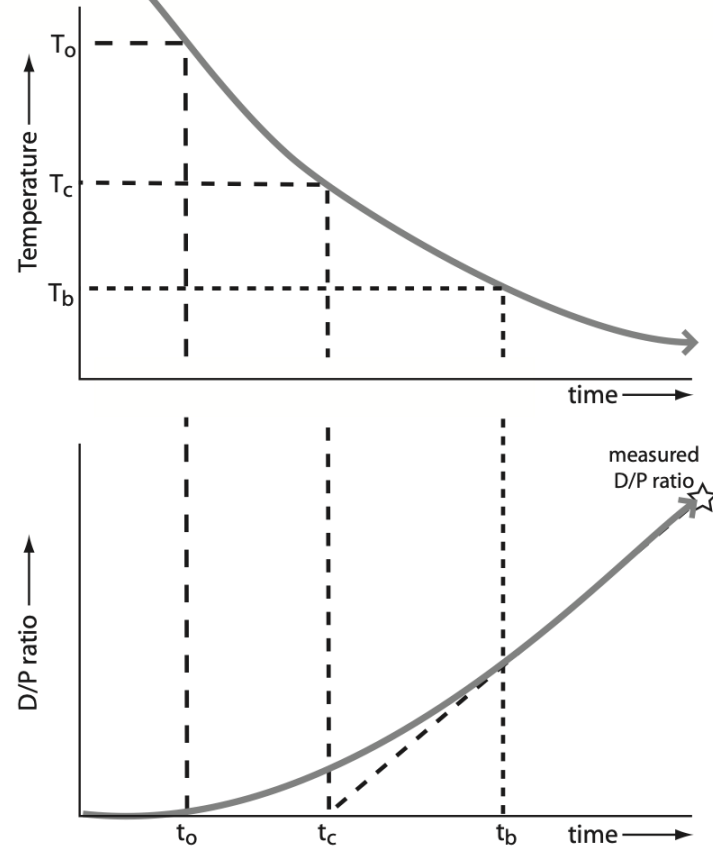
where D_0 is the diffusivity at infinite temperature (diffusion constant), a is the diffusion domain, E_a is the activation energy, R is the gas constant and T_K is temperature in Kelvins

- The **diffusion domain** a is typically the size of the mineral itself (simple systems)
- The **activation energy** E_a is the minimum energy that must be put into the system for diffusion to occur



Temperature-dependent diffusion

Fig 2.1, Braun et al., 2006



- The 'open system' temperature T_o
The time/temperature that corresponds to the lower limit to the fully open system
- The closure temperature T_c
The temperature of the system at the time corresponding to its age (Dodson)
- The blocking temperature T_b
The upper temperature limit of fully closed system behavior



Dodson's effective closure temperature

- **Method for calculating the closure temperature of a thermochronological system based on the observed diffusion parameters and the rock/mineral cooling rate (Dodson, 1973)**
- Assumption: In the partial retention zone, temperature varies as the inverse of time ($T \propto 1/t$)
- Approximate solution to the temperature-dependent diffusion equation with a **diffusivity**

$$D(t) = D(0)e^{-t/\tau}$$

where τ is the time taken for the diffusivity to decrease by a factor of 1/e



Dodson's effective closure temperature

- After some mathematical manipulation we can solve for τ and find

$$\tau = -\frac{RT^2}{E_a \dot{T}}$$

where \dot{T} is the cooling rate (negative by convention)

- Dodson's closure temperature equation is

$$T_c = \frac{E_a}{R \ln(A\tau D_0/a^2)}$$

where A is a geometry factor (25 for a sphere, 27 for a cylinder and 8.7 for a plane sheet)

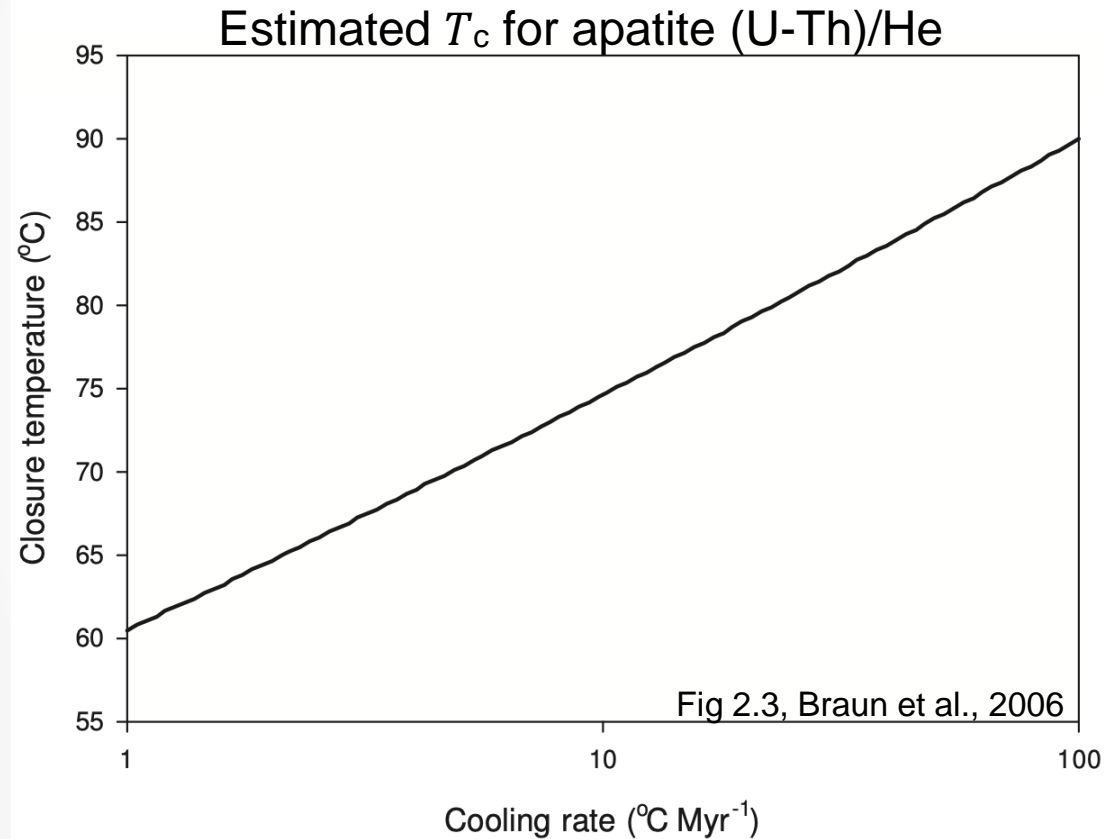


Pseudo-code for solving Dodson's equation

- Define constants
- Define initial “guess” for value of T_c
- Loop over some range to iterate on values of τ and T_c
 - Calculate new τ with current value of T_c
 - Calculate new value of T_c for new τ value
 - Check to see how much value of T_c has changed since last iteration
 - If value has not changed more than some very small number, exit loop and output calculated ‘final’ T_c value



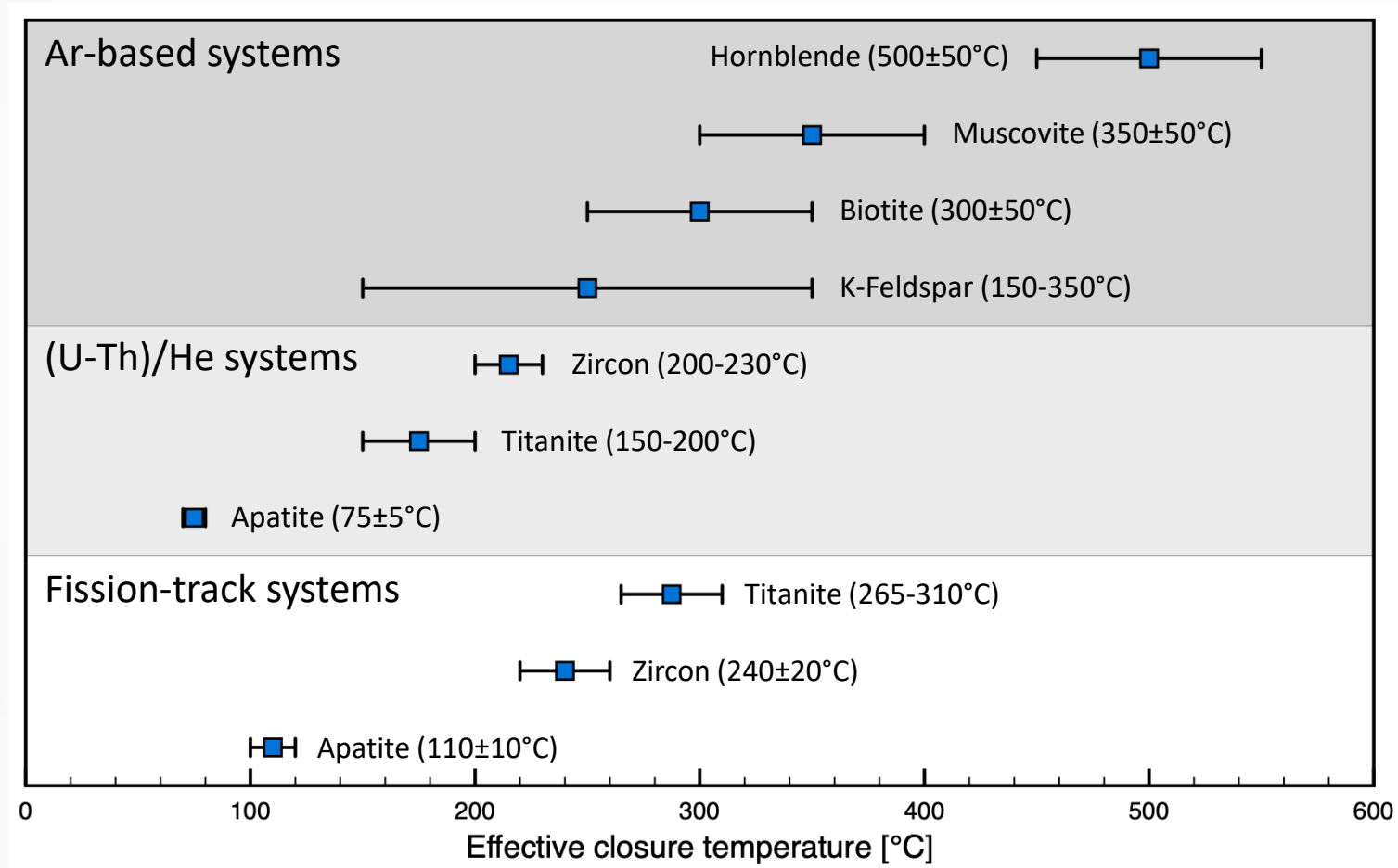
Dodson's effective closure temperature



The effective closure temperature T_c increases significantly at higher cooling rates



Common thermochronometers





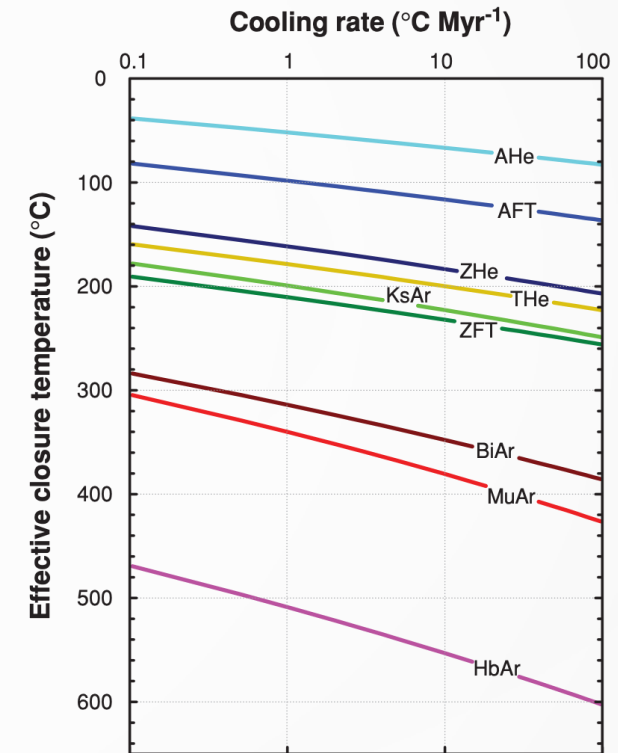
Main takeaways

- What is the basic idea for thermochronology?
- What is an effective closure temperature and how does it relate to the rate of cooling of a mineral sample?



Exercise 3: Diffusion and Dodson's equation

- You will...
 - Play around with diffusion
 - Calculate closure temperatures
 - Explore Dodson's equation
- **Due: 19 Nov. at 12:15**



Reiners and Brandon, 2006



References

- Ault, A. K., Gautheron, C., & King, G. E. (2019). Innovations in (U–Th)/He, Fission Track, and Trapped Charge Thermochronometry with Applications to Earthquakes, Weathering, Surface-Mantle Connections, and the Growth and Decay of Mountains. *Tectonics*, 38(11), 3705–3739. <https://doi.org/10.1029/2018TC005312>
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- Dodson, M. H. (1973). Closure temperature in cooling geochronological and petrological systems. *Contributions to Mineralogy and Petrology*, 40(3), 259–274. <https://doi.org/10.1007/BF00373790>
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- Valla, P. G., Beek, P. A. Van Der, & Braun, J. (2011). *Rethinking low-temperature thermochronology data sampling strategies for quantification of denudation and relief histories: A case study in the French western Alps*. 307, 309–322. <https://doi.org/10.1016/j.epsl.2011.05.003>