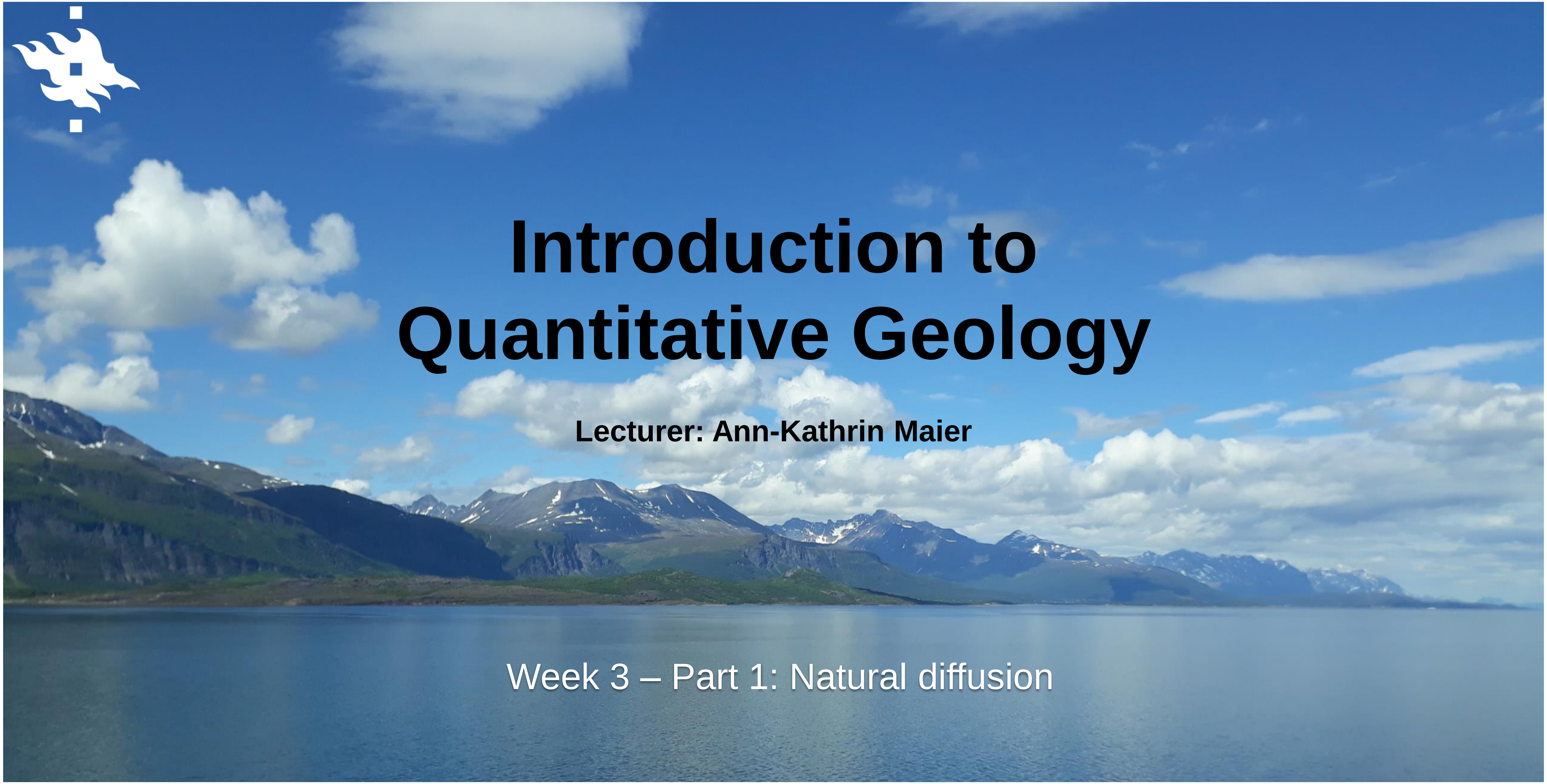


Introduction to Quantitative Geology

Lecturer: Ann-Kathrin Maier

Week 3 – Part 1: Natural diffusion

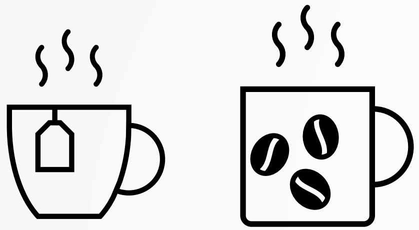
Last week

- We talked about comparing predictions to observations:
  - Linear regression
  - Linear correlation
  - Goodness-of-fit

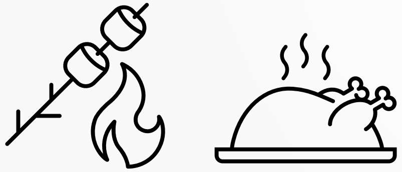
This week

- **Part 1: Natural diffusion**
  - General concepts of diffusion
  - Mathematical definition
- **Part 2: Basic concepts of thermochronology**
  - What is it?
  - General terms and concepts

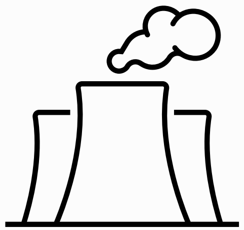
Diffusion in daily life



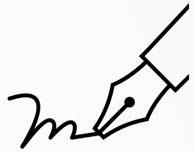
Brewing tea/coffee



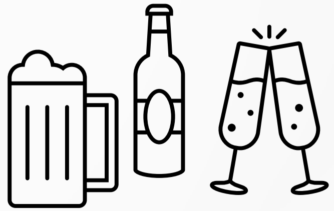
Heat in a solid (conduction)



Smoke in the air



Ink on paper



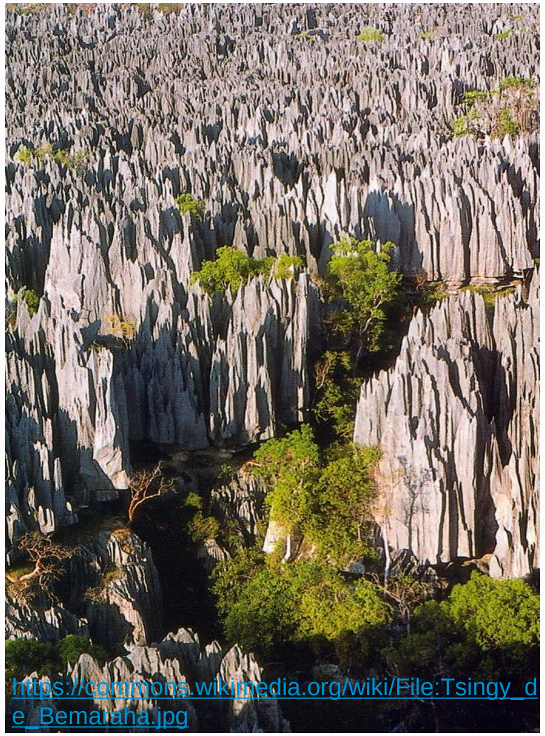
CO_2 in a carbonated beverage

Diffusion as a geological process: Landscapes

https://commons.wikimedia.org/wiki/File:AS17-14021493%2BAS17-140-21497.jpg

Moon surface (Eugene Cernan, 1972)

diffusion-dominated

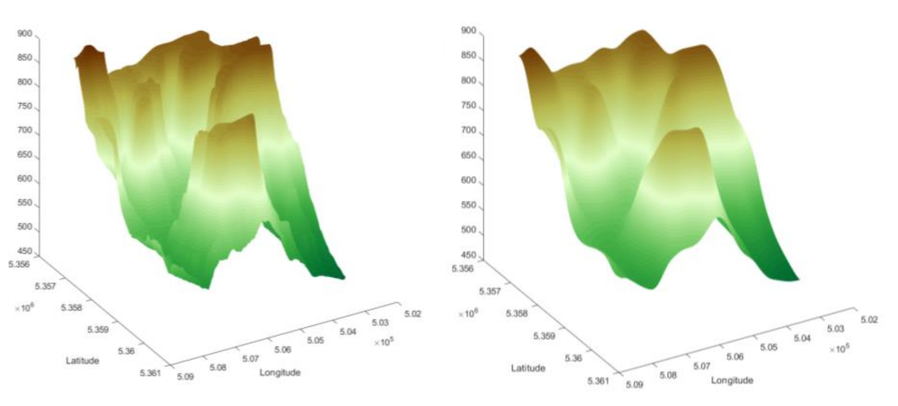


https://commons.wikimedia.org/wiki/File:Tsingy_de_Bemaraha.jpg

Tsingy de Bemaraha, Madagascar

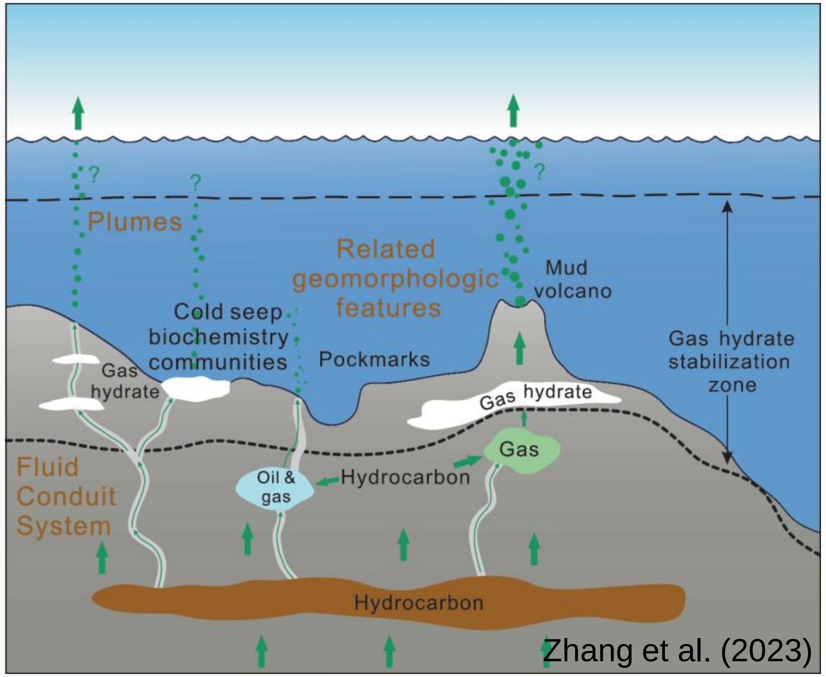
dissolution-dominated

https://commons.wikimedia.org/wiki/File:Moessingen-Rutschung-gross.Jpg

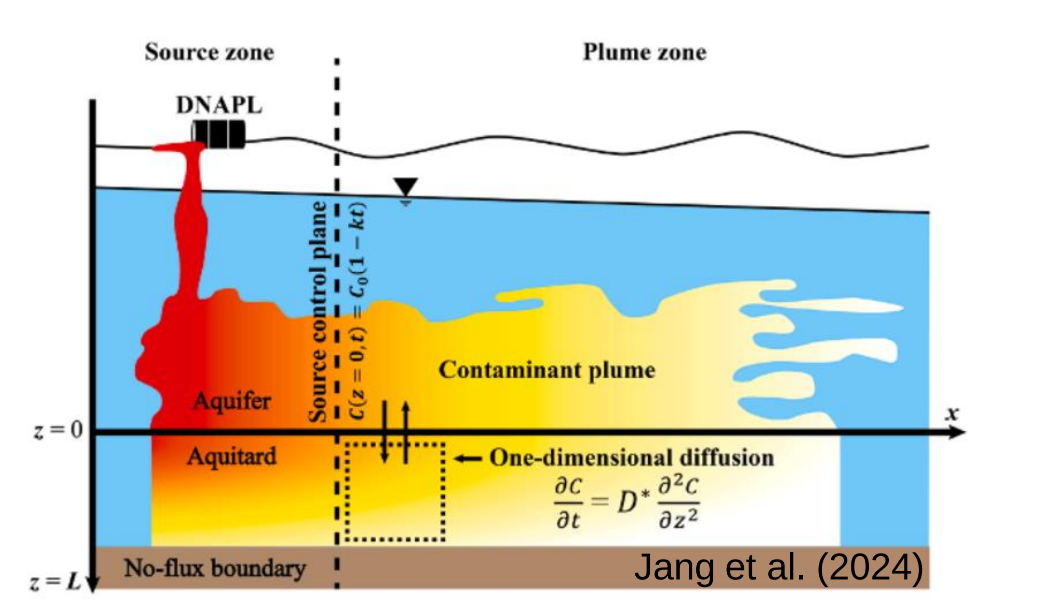


1 Myr of hillslope diffusion

Diffusion as a geological process: Subsurface

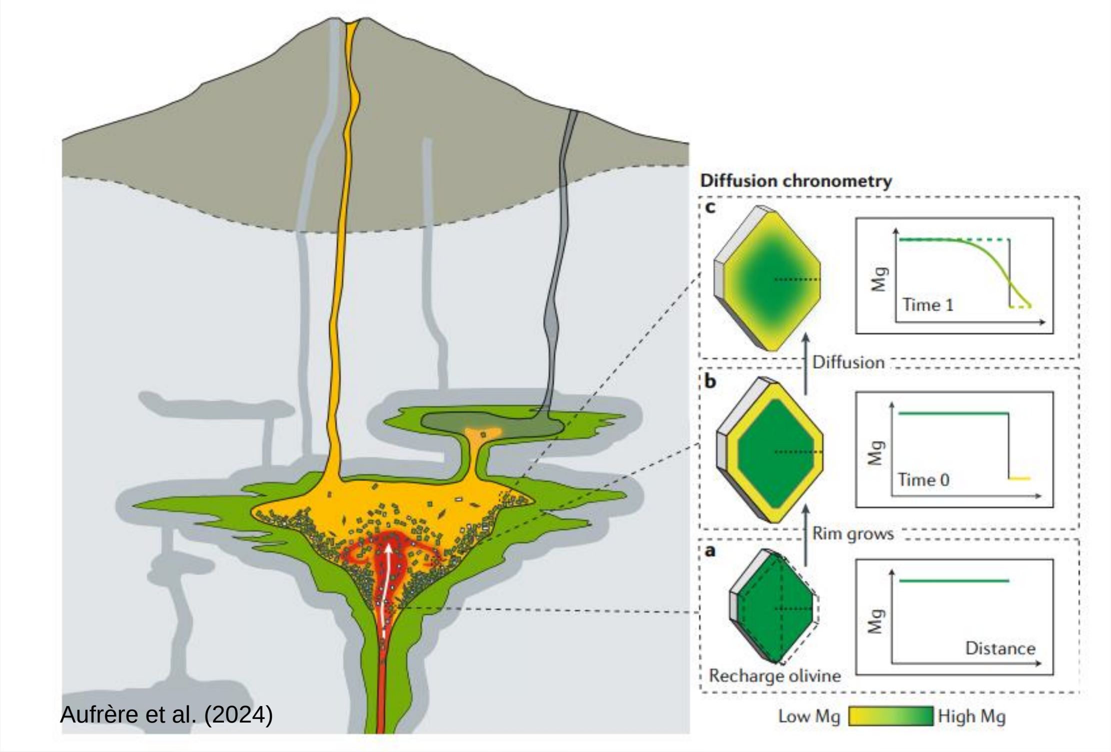


Gas seepage and migration



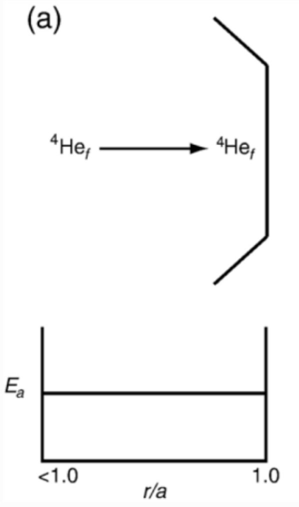
Pollutants in aquifers

Diffusion as a geological process: Mineral grains



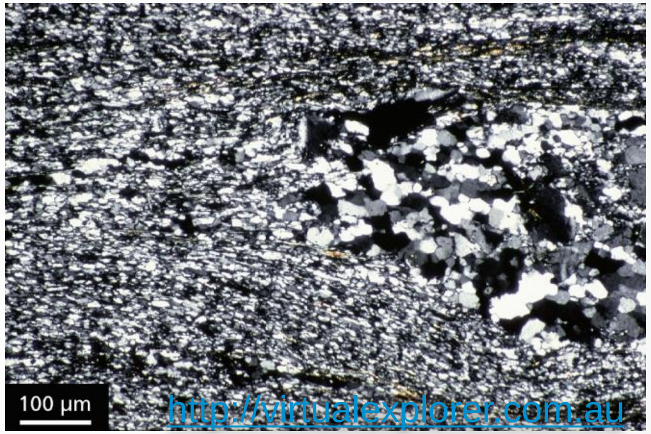
Aufrère et al. (2024)

Olivine diffusion chronometry



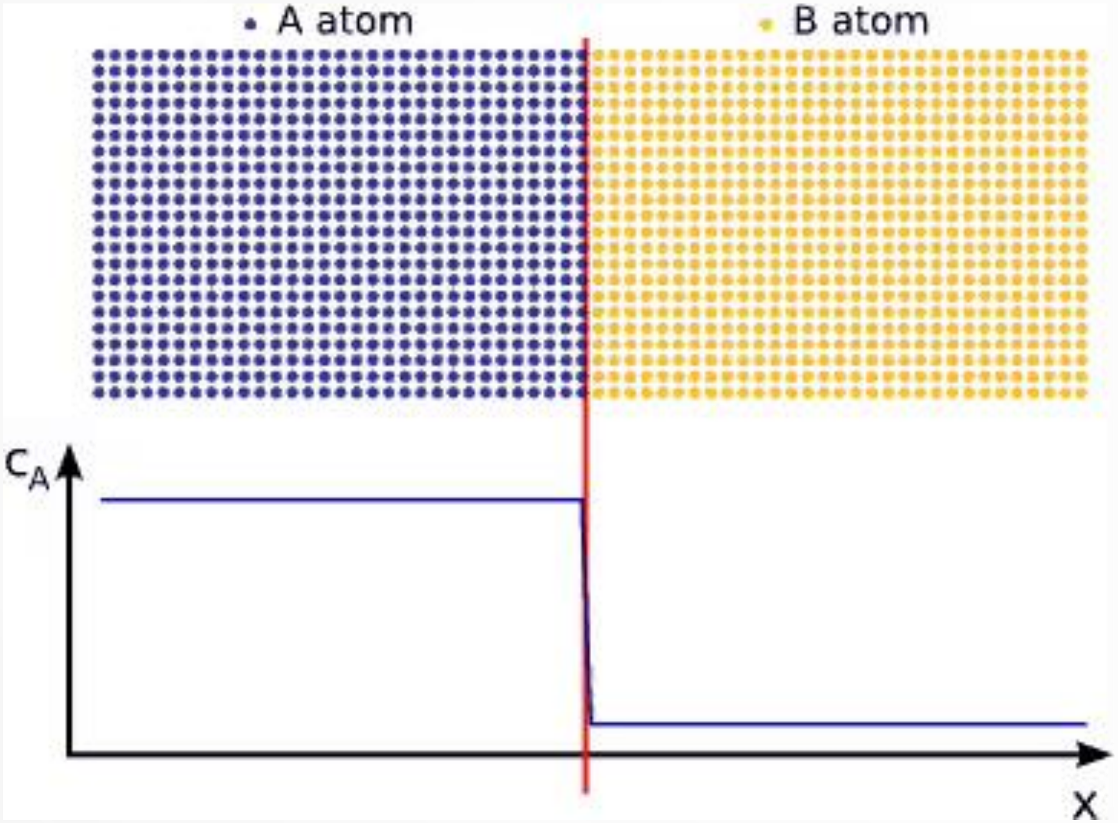
Shuster et al. (2006)

Diffusion of ^{4}He in an apatite crystal



Grain boundary sliding

The diffusion process



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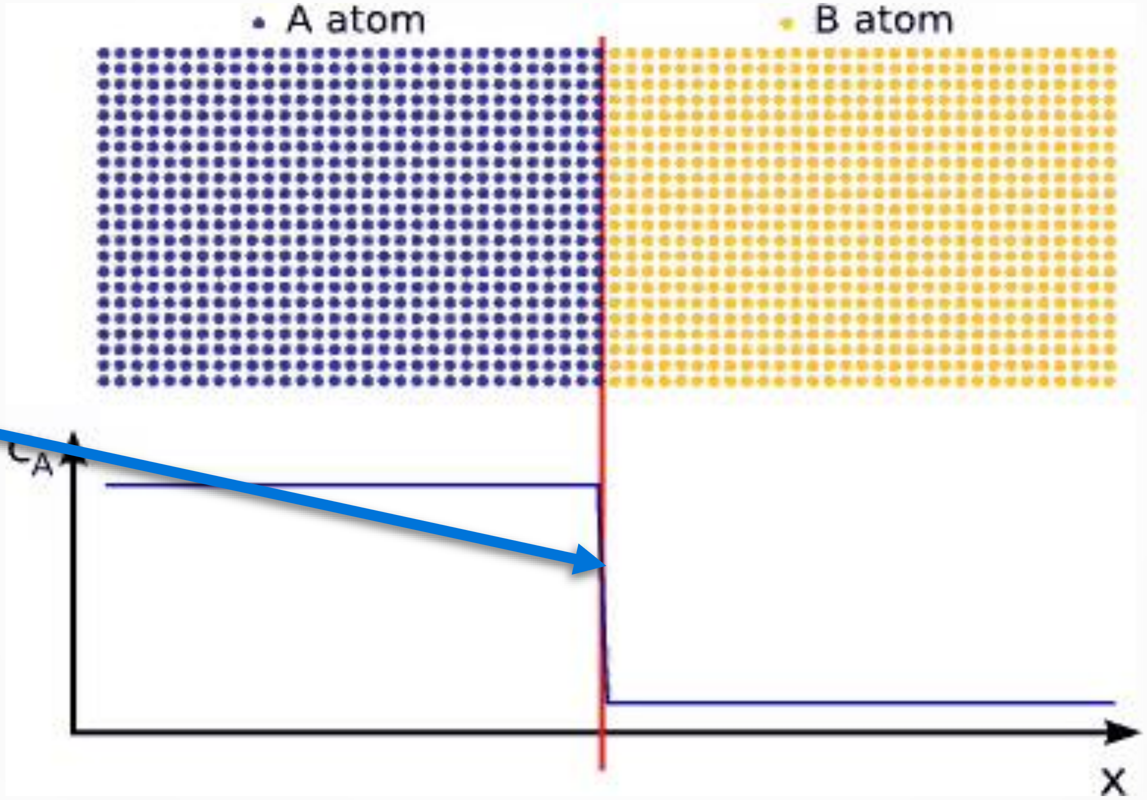
Diffusion:

Process in which the random motion of particles results in mass transport or mixing

The diffusion process

Concentration gradient



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Describing diffusion

- Process in which the **random motion** of particles results in mass transport or mixing
- Net motion from regions of high concentration to regions of low concentration
- Leads to **reduction concentration gradients**

Describing diffusion

- **Diffusion** occurs when a **conservative property** moves through space at a rate **proportional to a gradient**
- **Conservative property:** A quantity that must be conserved in the system (e.g., mass, energy, momentum)
- **Rate proportional to a gradient:** Movement occurs in direct relationship to the change in concentration

Mathematical definition

A mathematical definition

- **Diffusion** occurs when

  1) a **conservative property** moves through space

  2) at a rate **proportional to a gradient**

- From 2) :

  [The transportation rate] is proportional to [the change in concentration over some distance]

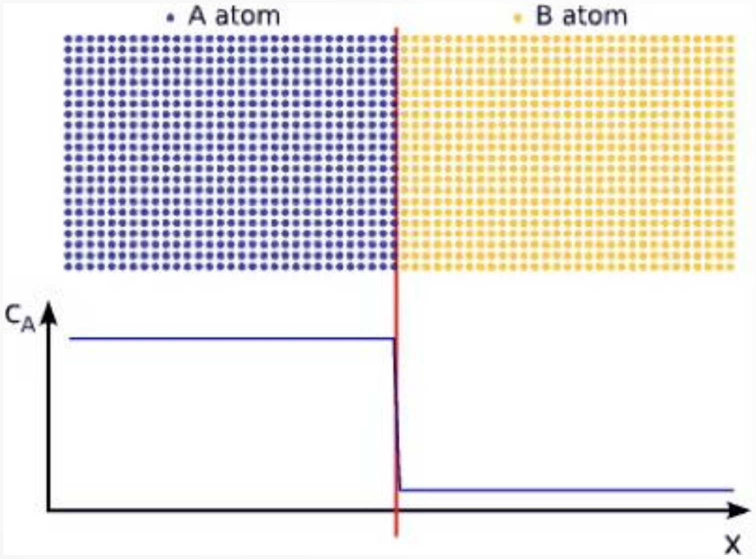
A mathematical definition

- In more quantitative terms: [The flux] is proportional to [the concentration gradient].
- As an equation:

$$q \propto \frac{\Delta C}{\Delta x}$$

where q is the (mass) flux, $\propto$ is the “proportional to” symbol, Δ indicates a change in the quantity that follows, C is the concentration and x is distance

A mathematical definition



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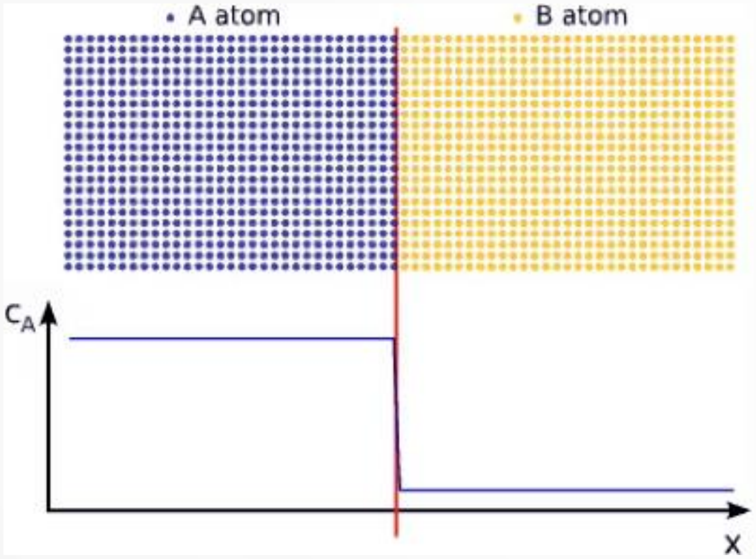
- If transport is directly proportional to the gradient, we can replace the proportional to symbol with a constant D (diffusion coefficient or diffusivity)
- We can also replace the finite changes Δ with infinitesimal changes ∂

$$q \propto \frac{\Delta C}{\Delta x} \quad \rightarrow \quad q = -D\frac{\delta C}{\delta x}$$

A mathematical definition



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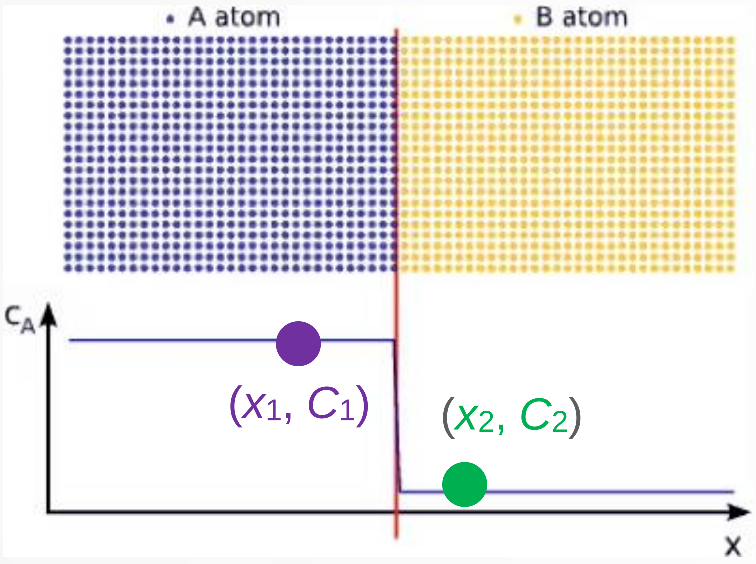
- Consider the example to the left:
  - Here, we can formulate the diffusion of atoms of A across the red line with time as

$$q = -D \frac{\delta C_A}{\delta x}$$

where C_A is the concentration of atoms of A.

A mathematical definition



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- OK, but **why is there a minus sign?**
- We can consider a simple case for finite changes at two points: (x_1, C_1) and (x_2, C_2)
- At those points, we could say

$$q = -D \frac{\Delta C}{\Delta x}$$

$$= -D \frac{C_2 - C_1}{x_2 - x_1}$$

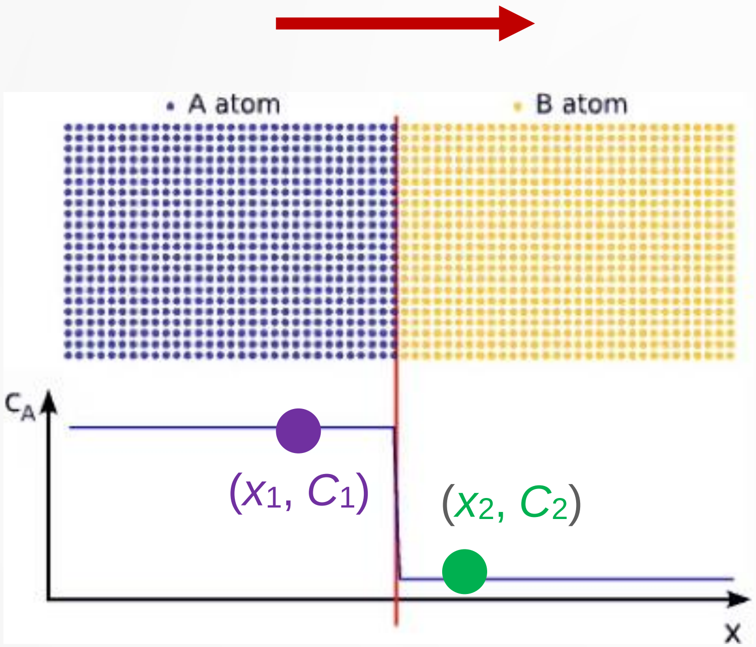
ΔC is negative

Δx is positive

The gradient is negative!

A mathematical definition

Positive flux of A



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- Multiplying the negative gradient by -D yields a positive flux q along the x axis

$$q = -D \frac{\delta C_A}{\delta x}$$

$$q = -D \frac{\Delta C}{\Delta x}$$

$$= -D \frac{C_2 - C_1}{x_2 - x_1}$$

A mathematical definition

- **Diffusion** occurs when
  1) a **conservative property** moves through space
  2) at a rate **proportional to a gradient**
- From 2) we got

$$q = -D \frac{\delta C_A}{\delta x}$$

where q is the (mass) flux, D ist the diffusion coefficient, C is the concentration and x is distance

A mathematical definition

- What about “ 1) a **conservative property moves through space”** ?

- We can say:

  [change in concentration with time] is equal to [change in transport rate with distance]

- In slightly more quantitative terms:

  [rate of change of concentration] is equal to [flux gradient]

A mathematical definition

- [rate of change of concentration] is equal to [flux gradient]

- As an equation:

$$\frac{\Delta C}{\Delta t} = -\frac{\Delta q}{\Delta x}$$

← conservation of mass/energy

where t is time

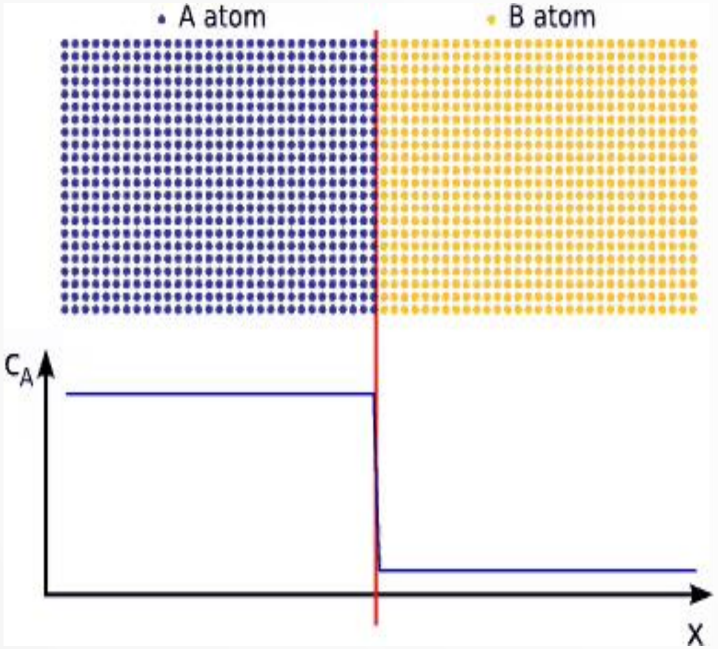
A mathematical definition

- How is $\frac{\Delta C}{\Delta t} = -\frac{\Delta q}{\Delta x}$ a conservation of mass/energy equation?
- Consider the fluxes q_1 and q_2 at two points, x_1 and x_2

$$\frac{\Delta C}{\Delta t} = -\frac{q_2 - q_1}{x_2 - x_1}$$

What happens when the flux of mass q_2 at x_2 is larger than the flux q_1 at x_1?

A mathematical definition



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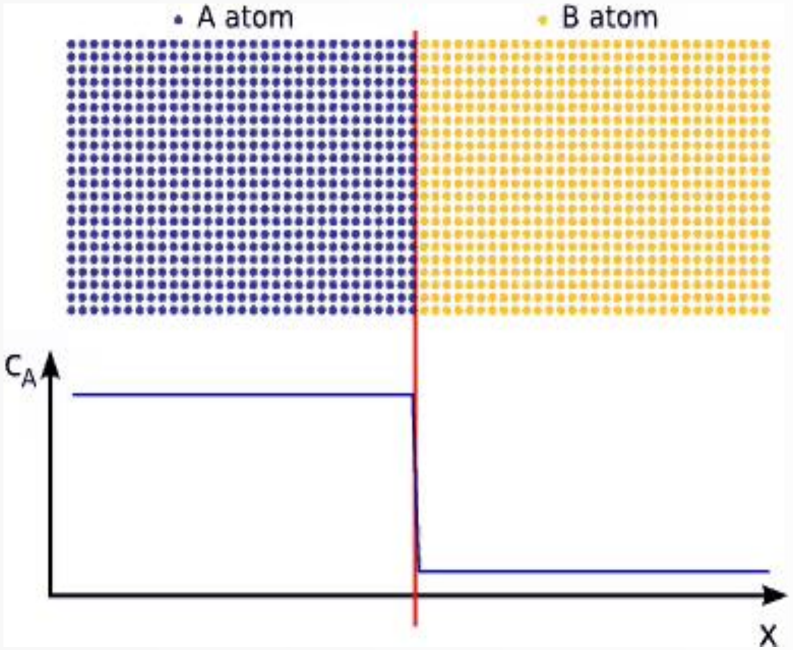
- Replace the finite changes Δ with infinitesimal changes ∂, we can describe our example on the left

$$\frac{\delta C}{\delta t} = -\frac{\delta q}{\delta x}$$

→ The concentration of A will change based on the flux across a reference face at position x minus the flux across a reference face at position $x + dx$

A mathematical definition



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- In the exercise for this week we will explore some simple models for diffusion and some of the factors we need to consider

10 min break!

Up next: An introduction to Thermochronology

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