Introduction to Quantitative Geology

Rock and ice as viscous materials

Lecturer: David Whipp
david.whipp@helsinki.fi

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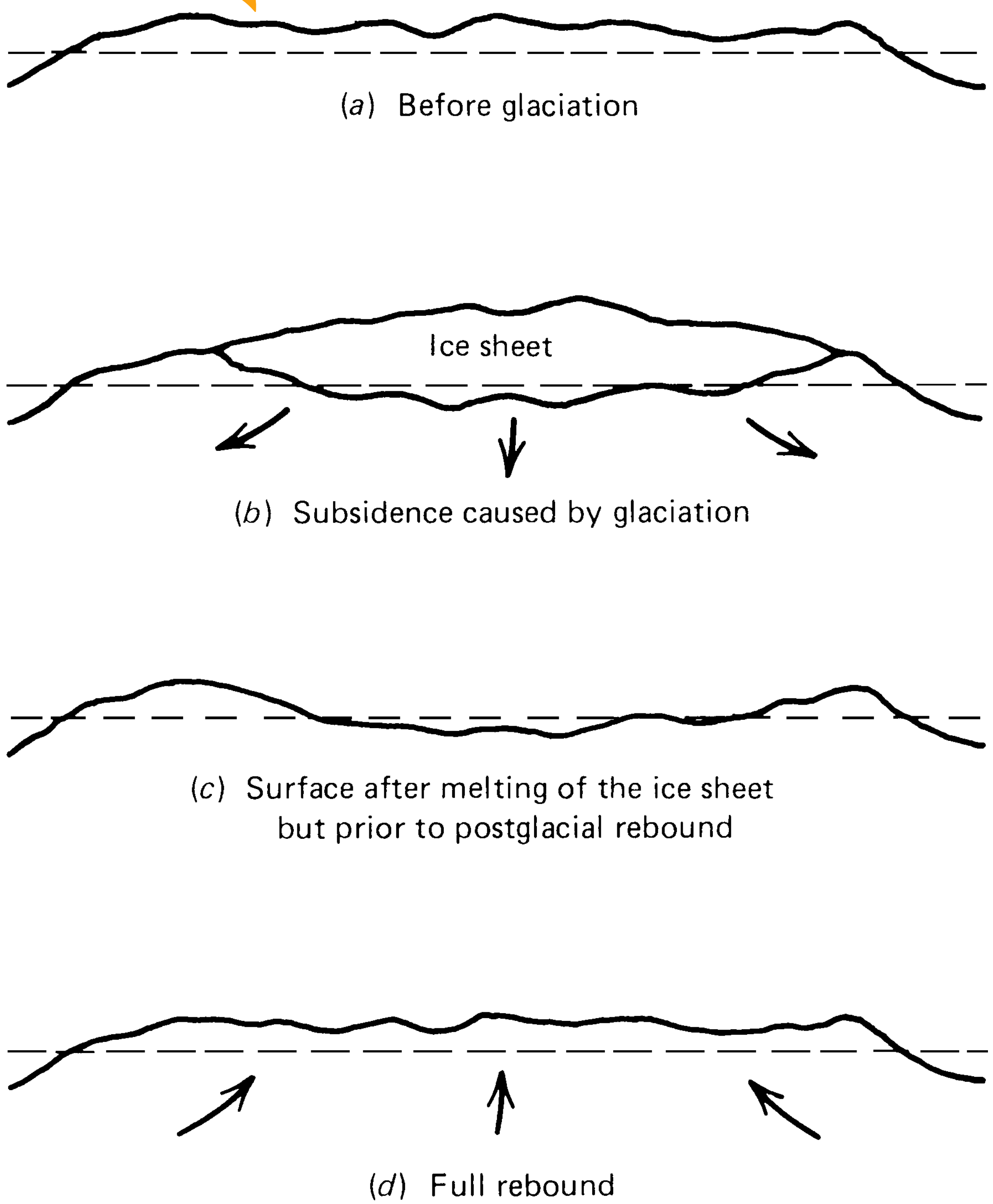
Goals of this lecture

- Introduce the basic relationship for **viscous flow** of rock and ice
- Explore two different end-member types of **viscous flow in a channel**
- Discuss the effects of **temperature on viscosity** and **nonlinear viscosity**

Examples of viscous flow: Alpine glaciers

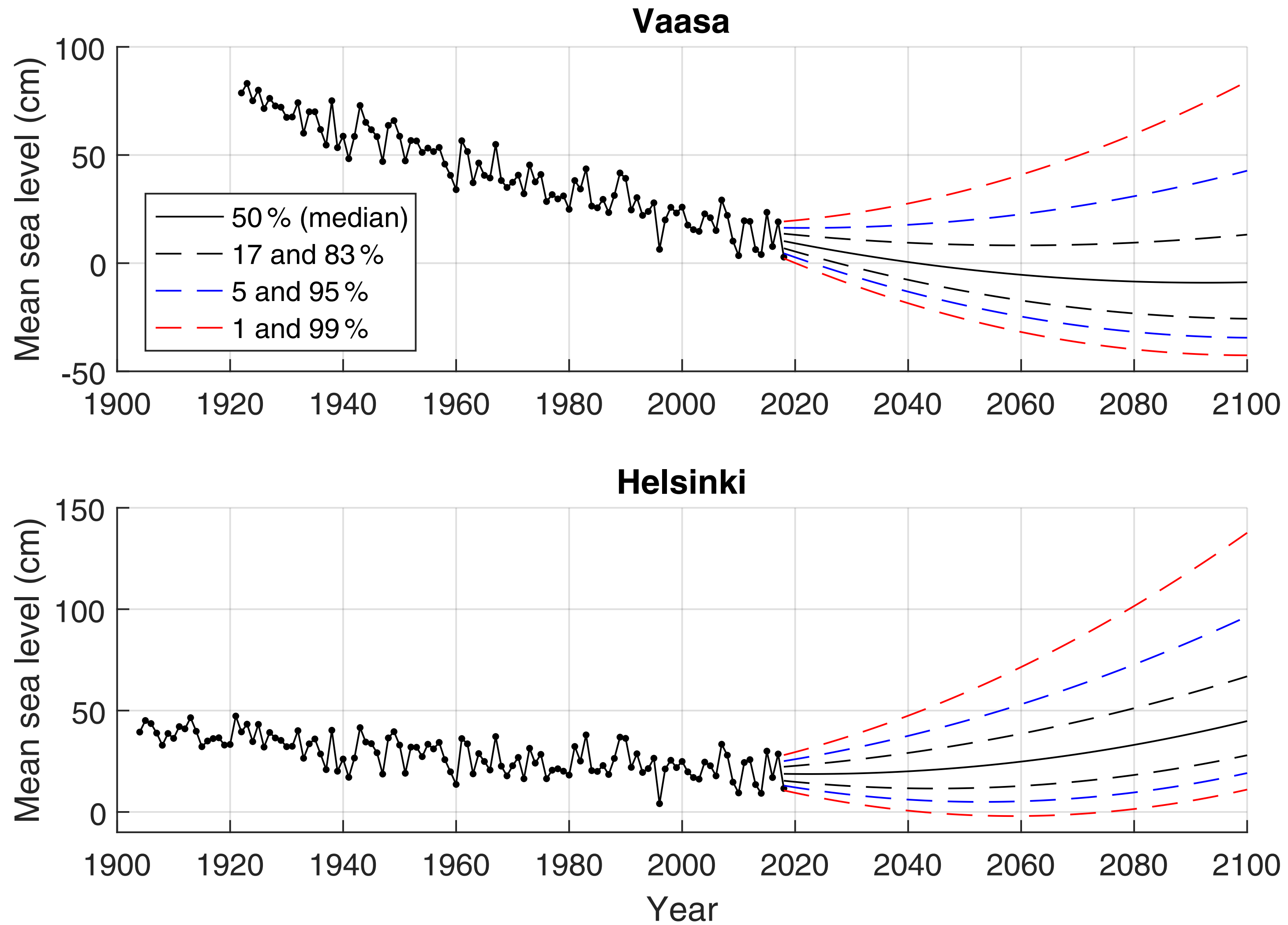
- **Alpine glaciers flow downhill** under their own weight

Glacial isostatic adjustment



Turcotte and Schubert, 2014

Pellikka et al., 2023



Surface uplift due to glacial isostatic adjustment is controlled by **flow of the underlying asthenosphere**

What is a fluid?

- **Fluid**: Any material that flows in response to an applied stress
  - Deformation is continuous
  - Stress is proportional to strain rate

$$\tau \propto \frac{du}{dz}$$

where τ is the shear stress, du/dz is the velocity gradient (equivalent to strain rate) and u is the velocity in the x-direction

Viscosity, defined

http://en.wikipedia.org

- Constant of proportionality η is known as the **dynamic viscosity**, or often simply **viscosity**

$$\text{1-D:} \qquad \tau = \eta \frac{du}{dz}$$

- **Viscosity** has units of Pa s (Pascal seconds) or kg m^{-1} s^{-1}
- You can think of viscosity as a resistance to flow
  - Higher viscosity → more resistant to flow, and vice versa
- The terms kinematic viscosity and bulk viscosity (or compressibility) are not the same thing as the dynamic viscosity

Approximate viscosities of common materials

Material	Viscosity [Pa s]
Air	10^{-5}
Water	10^{-3}
Honey	10^{1}
Basaltic lava	10^{3}
Ice	10^{10}
Rhyolite lava	10^{12}
Rock salt	10^{17}
Granite	10^{20}

A honey dipper works because of the viscosity of honey

- Viscosity of natural materials is hugely variable
    - Range of almost 20 orders of magnitude for rocks and lava

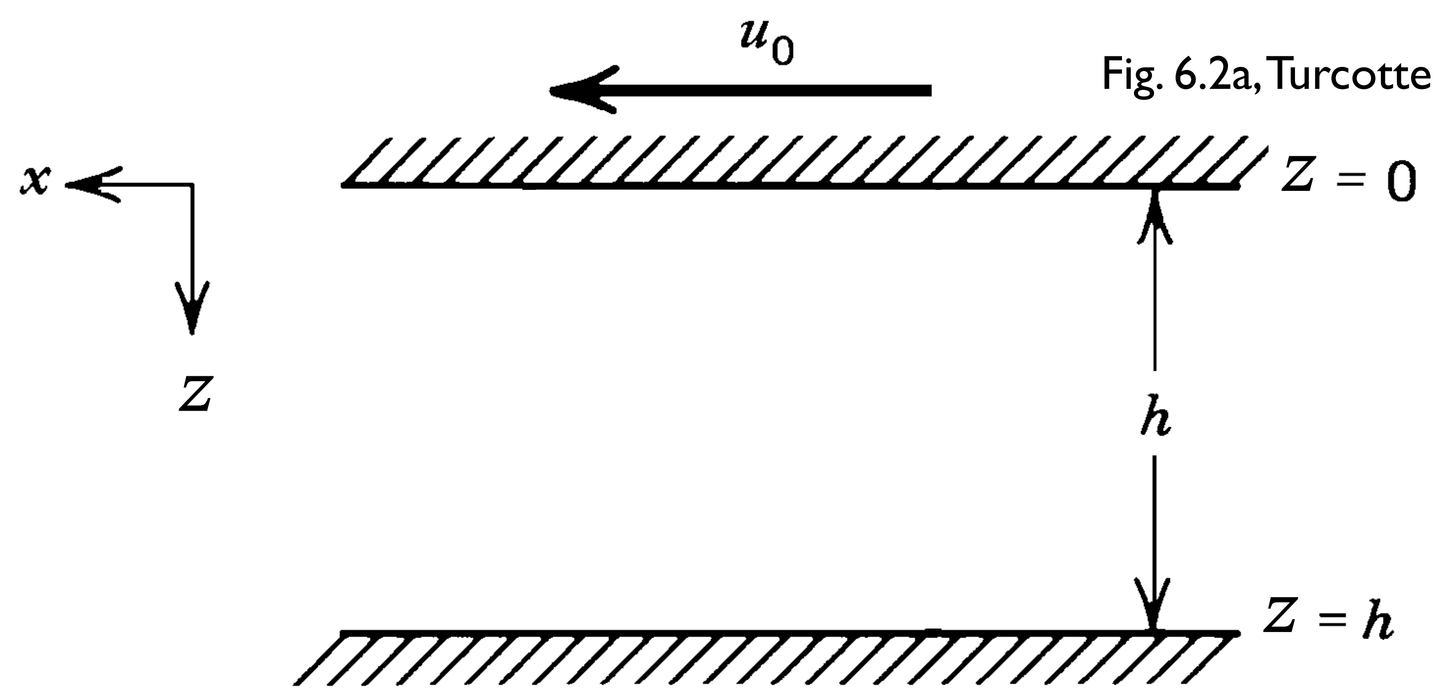
Newtonian (linear) viscosity

$$\tau = \eta \frac{du}{dz}$$

- A **Newtonian material** has a linear relationship between shear stress and strain rate
  - In other words, η is a constant value that does not depend on the stress state or flow velocity
- Air, water and thin motor oil are practically Newtonian fluids
- Rocks rarely deform as Newtonian fluids

Linear viscous flow in a channel

Fig. 6.2a, Turcotte and Schubert, 2014



- The general solution for the 1-D velocity of a fluid across a channel with boundary conditions (1) $u = 0$ at $z = h$ and (2) $u = u_0$ at $z = 0$ is

$$u = \frac{1}{2\eta}\frac{dp}{dx}(z^2 - hz) - \frac{u_0 z}{h} + u_0$$

where dp/dx is the applied pressure gradient

Styles of linear viscous flow: Couette flow

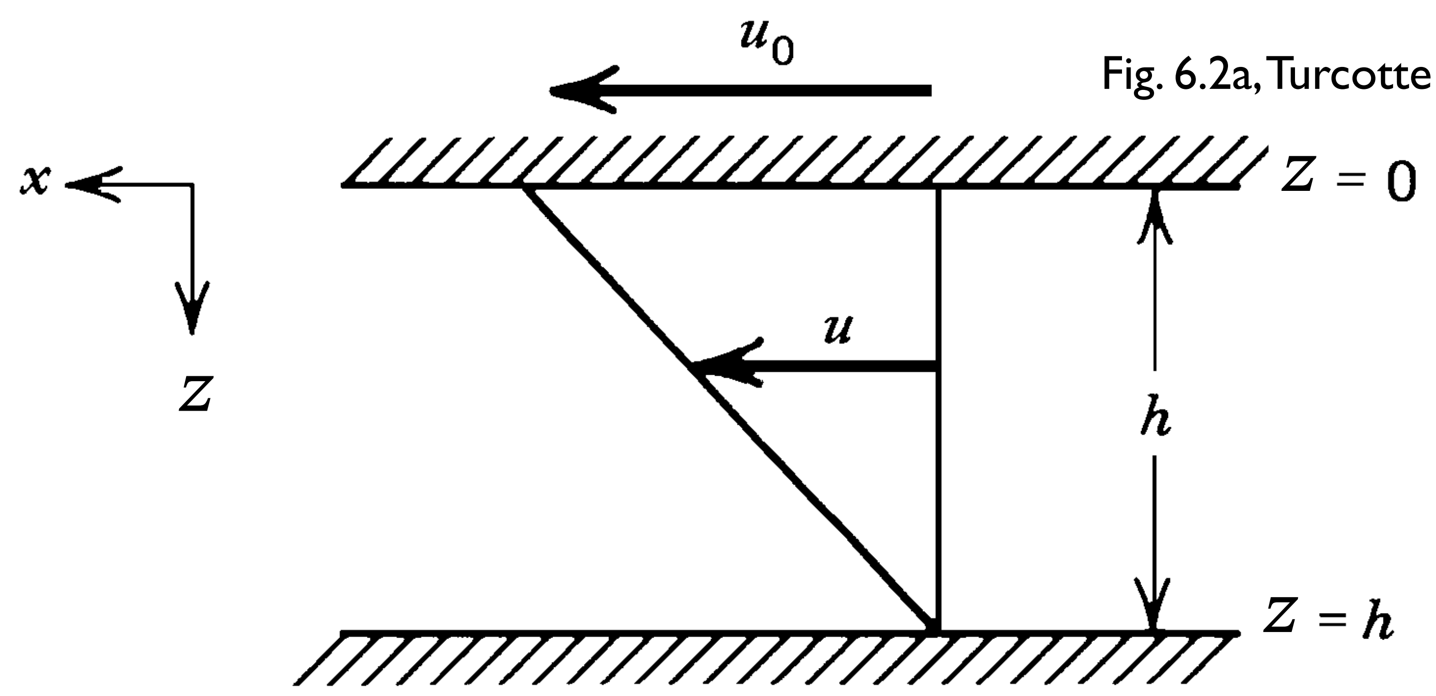
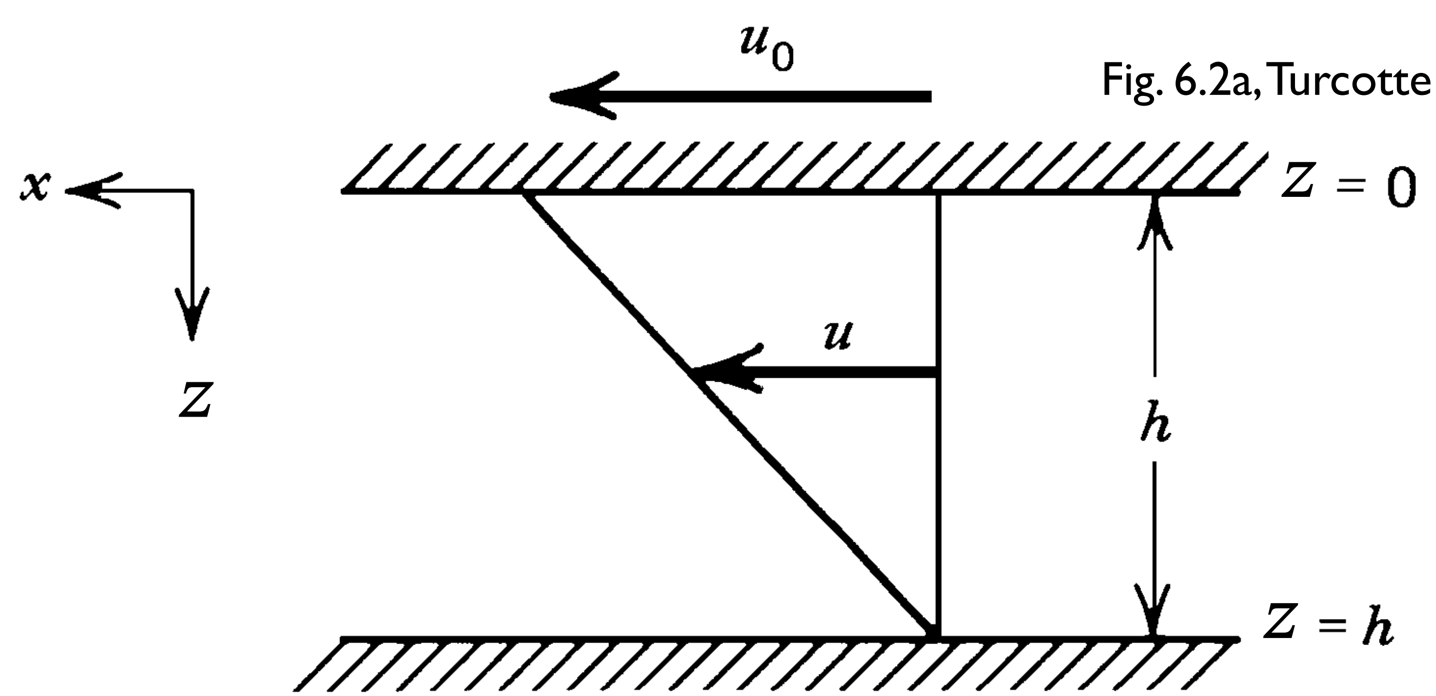


Fig. 6.2a, Turcotte and Schubert, 2002

(a) Couette flow $\frac{dp}{dx} = 0,\ u_0 \neq 0$

- **Couette flow** occurs when there is (1) a <u>difference in velocity between the channel boundaries</u> and (2) effectively <u>no pressure gradient</u>

Couette flow solution



(a) Couette flow $\frac{dp}{dx} = 0$, $u_0 \neq 0$

Fig. 6.2a, Turcotte and Schubert, 2002

- If we assume $dp/dx = 0$,

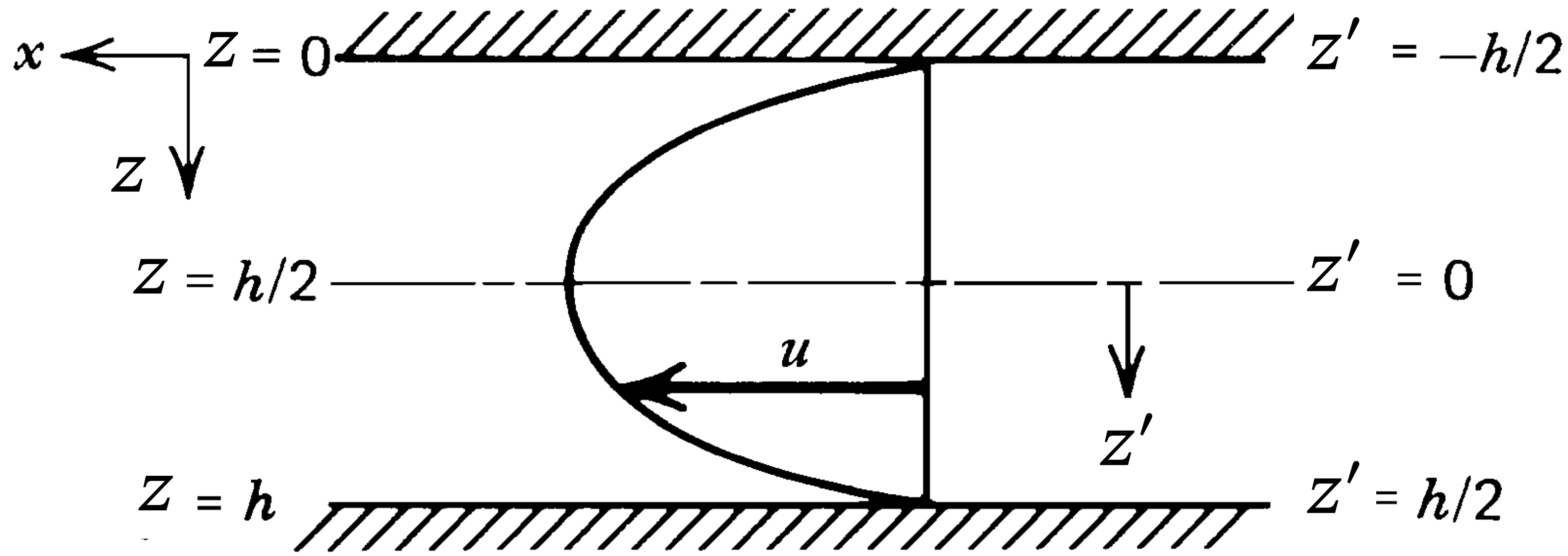
$$u = \frac{1}{2\eta}\frac{dp}{dx}(z^2 - hz) - \frac{u_0 z}{h} + u_0$$

reduces to

$$u = u_0\left(1 - \frac{z}{h}\right)$$

Poiseuille flow

Fig. 6.2b, Turcotte and Schubert, 2002

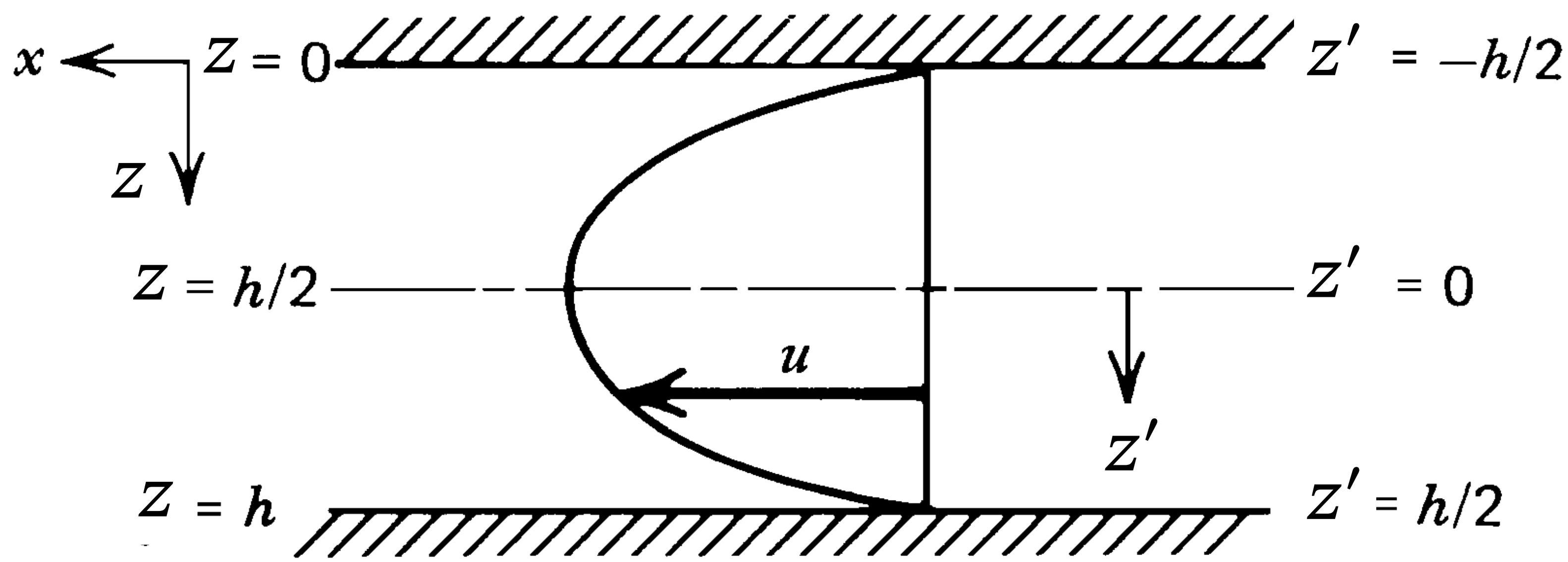


(b) $\frac{dp}{dx} \neq 0, u_0 = 0$

- **Poiseuille flow** occurs when (1) there is <u>no velocity difference between the walls of the channel</u> and (2) a <u>pressure gradient is applied</u>

Poiseuille flow solution

Fig. 6.2b, Turcotte and Schubert, 2002



(b) $\frac{dp}{dx} \neq 0,\ u_0 = 0$

- Using the same equation as we have previously, we can start with the general solution

$$u = \frac{1}{2\eta}\frac{dp}{dx}(z^2 - hz) - \frac{u_0 z}{h} + u_0$$

- If we set $u_0 = 0$, the velocity solution becomes

$$u = \frac{1}{2\eta}\frac{dp}{dx}(z^2 - hz)$$

Salt tectonics

- One example of a geological system that can exhibit both **Couette** and **Poiseuille** flow behavior is the flow of rock salt beneath sedimentary overburden

Temperature dependence

- In general, rock viscosity depends strongly temperature

$$\eta = A_0 \mathrm{e}^{Q/RT_\mathrm{K}}$$

where A_0 and Q are material properties known as the **pre-exponent constant** and **activation energy**, R is the universal gas constant and T_K is temperature in Kelvins

Temperature-dependent viscosity

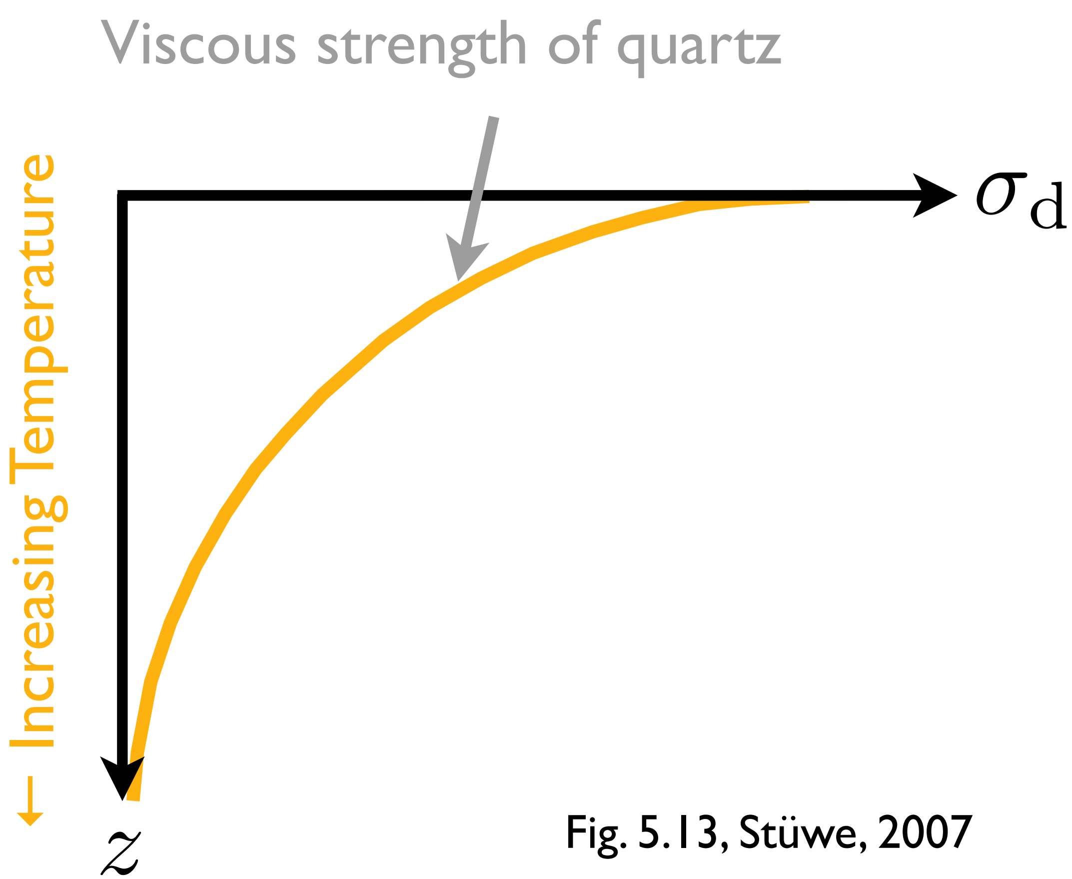


Fig. 5.13, Stüwe, 2007

- The viscous strength of quartz, for example, <u>rapidly decreases with increasing temperature</u>
- Note that the viscous strength is simply the <u>viscosity η multiplied by a nominal strain rate</u>

Temperature-dependent viscosity

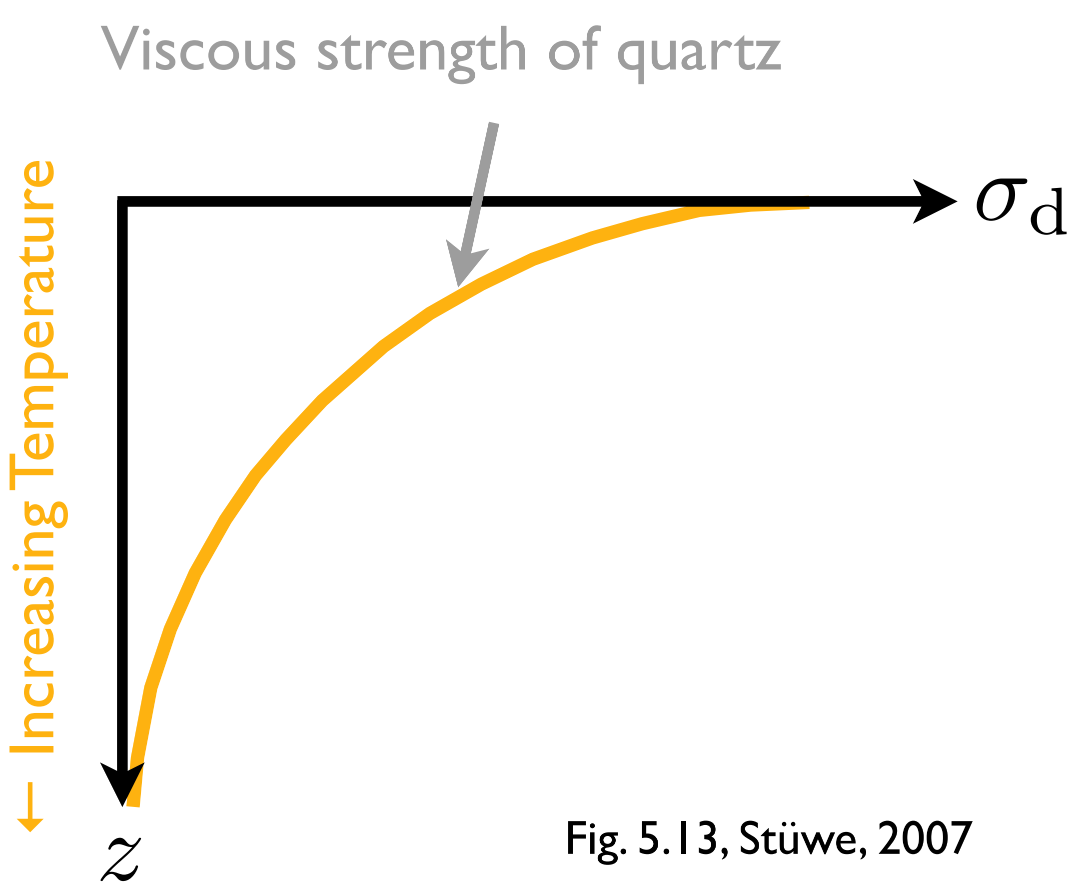


Fig. 5.13, Stüwe, 2007

- The viscous strength of quartz, for example, rapidly decreases with increasing temperature
- Note that the viscous strength is simply the viscosity η multiplied by a nominal strain rate
- **How might temperature-dependent viscosity be important in the Earth?**

Nonlinear viscosity

- In general, rocks will deform about 8 times as quickly when the applied force is doubled
    - Relationship between shear stress and strain rate is thus **NOT linear**
- Mathematically, we can say

$$\tau^n = A_{\text{eff}} \frac{du}{dz}$$

where n is the **power law exponent** and A_{eff} is a material constant

    - The power law exponent for many rocks is 2-4
    - A_{eff} is similar to η, but has units of Pa^n s

Flow of glaciers

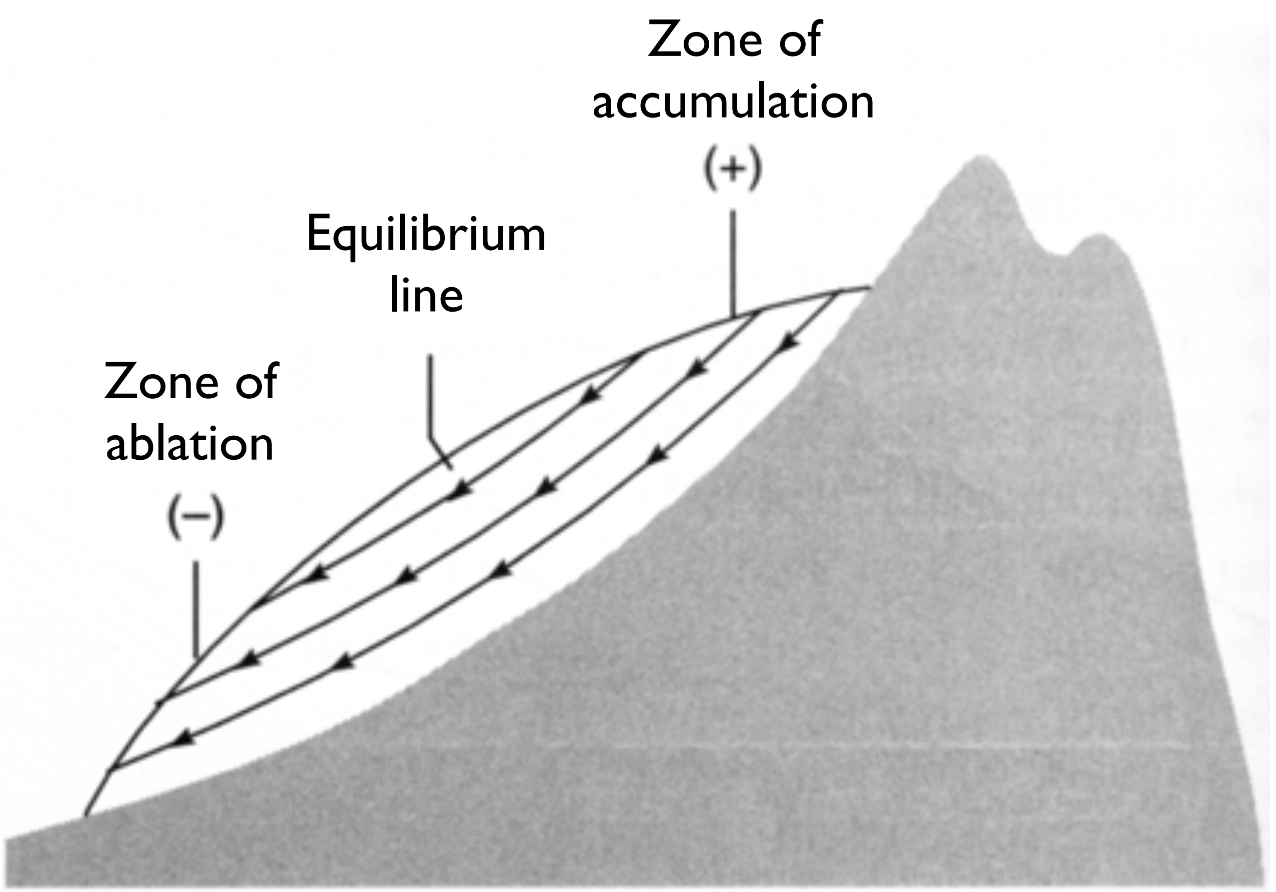


Fig. 9.14, Ritter et al., 2002

- Gravity drives the flow of alpine glaciers from higher elevation zones of **accumulation** to lower elevation zones of **ablation**
- Depending on the temperature of the region and the ice itself, the glacier may either be frozen to the bedrock (**cold-based**) or sliding along the bedrock (**warm-based**)

How do glaciers move?

Briksdal Glacier, Norway

- **Basal sliding**
  - Bottom of the glacier sliding along the substrate
  - Can occur as a result of slip atop a thin water layer, melting/re-freezing or slip atop water-saturated sediment
- **Internal deformation**
  - Ice flow is nonlinear viscous and sensitive to temperature
  - Deformation is concentrated near the bed

Flow of glaciers

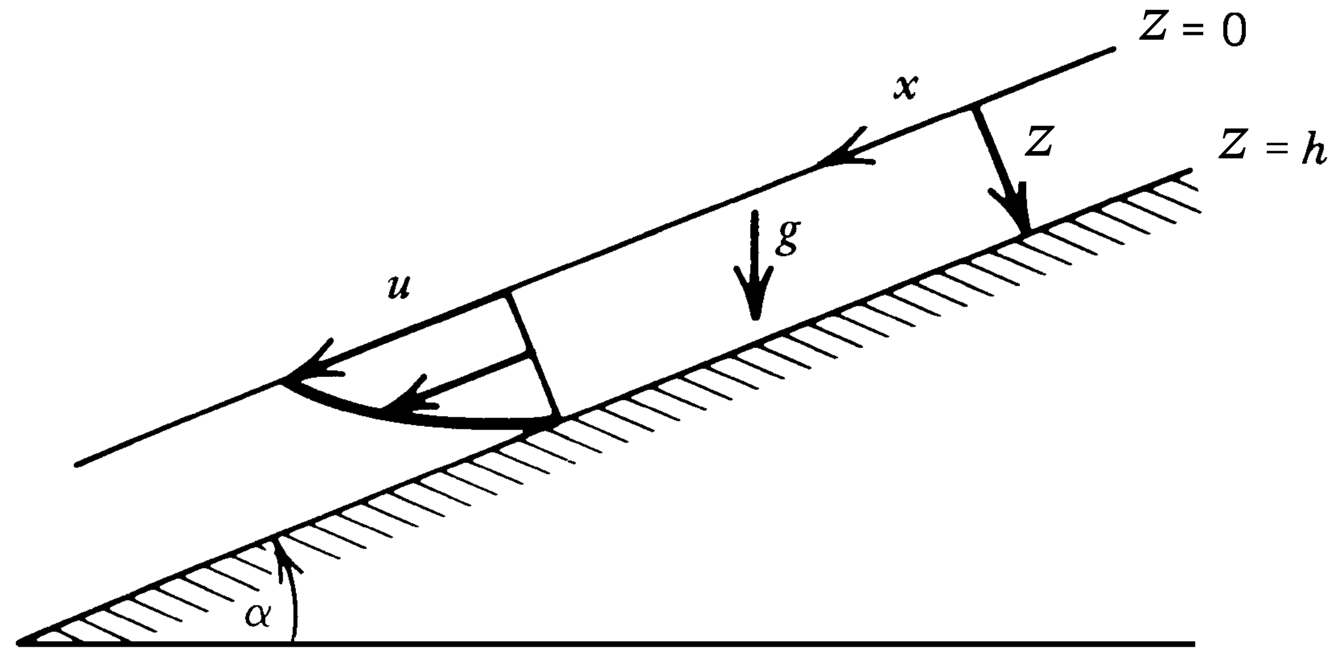


Fig. 6.3, Turcotte and Schubert, 2014

- In the exercise this week, we will look more closely at glacial flow
    - Velocity profile across a glacial valley
    - Down an incline

Recap

- **Viscous flow** is a common deformation behavior for rock and ice, where the deformation rate is proportional to the applied shear stress

- **Couette** and **Poiseuille** flows refer to end-member behaviors of linear viscous channel flows, and depend on the channel boundary velocities and pressure changes along the channel

- Most rocks do not exhibit a linear relationship between stress and strain rate (nonlinear viscosity), and their viscosity is strongly temperature-dependent

References

Pellikka, H., Johansson, M. M., Nordman, M., and Ruosteenoja, K.: Probabilistic projections and past trends of sea level rise in Finland, Nat. Hazards Earth Syst. Sci., 23, 1613–1630, https://doi.org/10.5194/nhess-23-1613-2023, 2023.

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Turcotte, D. L., & Schubert, G. (2014). *Geodynamics* (2nd ed.). Cambridge, UK: Cambridge University Press.