Introduction to Quantitative Geology

Geological advection:
Examples of advection in geological processes

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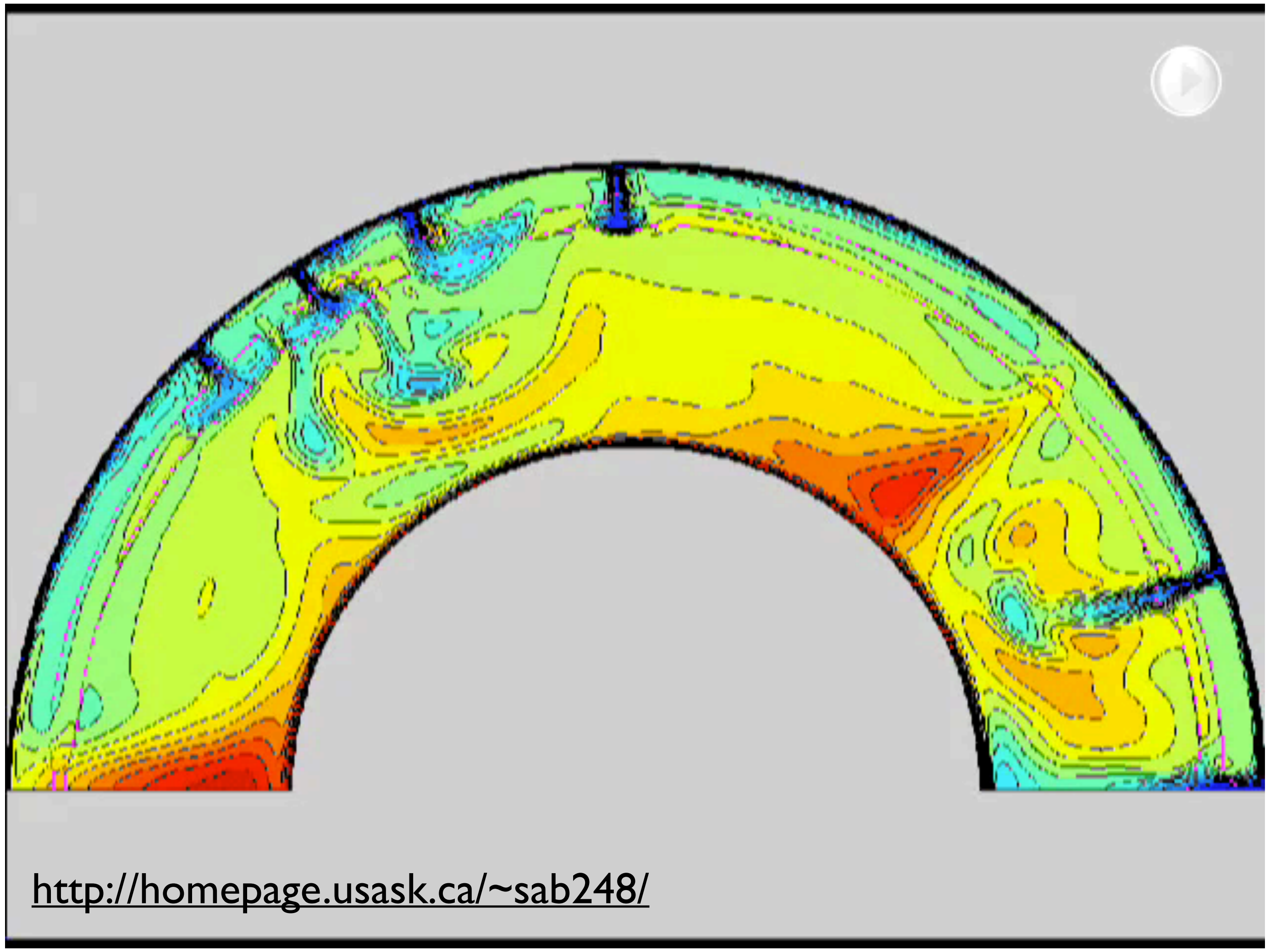
Goals of this lecture

- Introduce the **advection equation**
- Discuss an application of the advection equation to a **geological process**

What is advection?



- Advection involves a lateral translation of some quantity
    - In this example we see convection of heat, which is a combination of the transfer of heat by physical movement of molecules or atoms within a material and diffusion.

Diffusion equation

$$q = -D\frac{\partial C}{\partial x}$$

$$\frac{\partial C}{\partial t} = -\frac{\partial q}{\partial x}$$

- Last week we were introduced to the **diffusion equation**
  - Flux (transport of mass or transfer of energy) proportional to a gradient
  - Conservation of mass: Any change in flux results in a change in mass/energy

Diffusion equation

Diffusion

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$

$$q = -D\frac{\partial C}{\partial x}$$

$$\frac{\partial C}{\partial t} = -\frac{\partial q}{\partial x}$$

- Substitute the upper equation on the left into the lower to get the classic **diffusion equation**
  - q = flux per unit length

    D = diffusivity

    C = concentration

    x = distance

    t = time

Diffusion equation for heat transfer

Diffusion

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

- Substitute the upper equation on the left into the lower to get the classic **diffusion equation**
  - κ = thermal diffusivity
    T = temperature
    x = distance
    t = time

Advection and diffusion equations for heat transfer

Diffusion

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

Advection

$$\frac{\partial T}{\partial t} = v_z \frac{\partial T}{\partial z}$$

- This week we meet the **advection equation**

Advection and diffusion equations for heat transfer

Diffusion

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}$$

Advection

$$\frac{\partial T}{\partial t} = v_z \frac{\partial T}{\partial z}$$

- This week we meet the **advection equation**
- Two key differences:
  - Change in mass/energy with time proportional to gradient, rather than curvature (or *change* in gradient)
  - **Advection coefficient** v_z has units of $[L/T]$, rather than $[L^2/T]$

Advection and diffusion equations applied to river profiles

River channel profiles

Diffusion

$$\frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2}$$

Advection

$$\frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x}$$

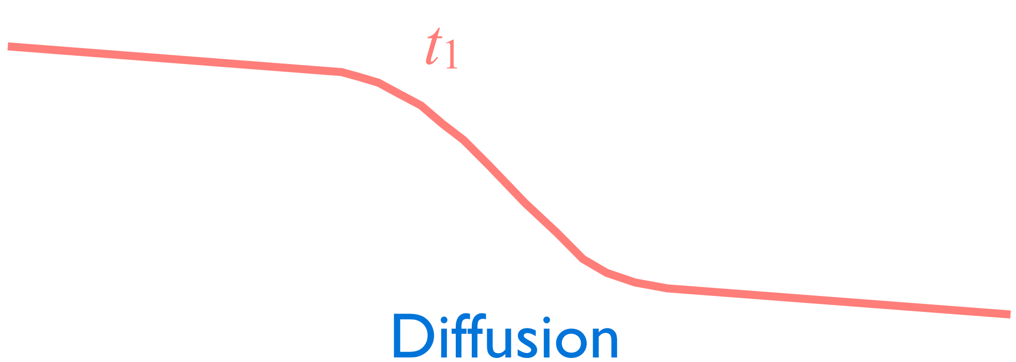


Fig. 1.7, Pelletier, 2008

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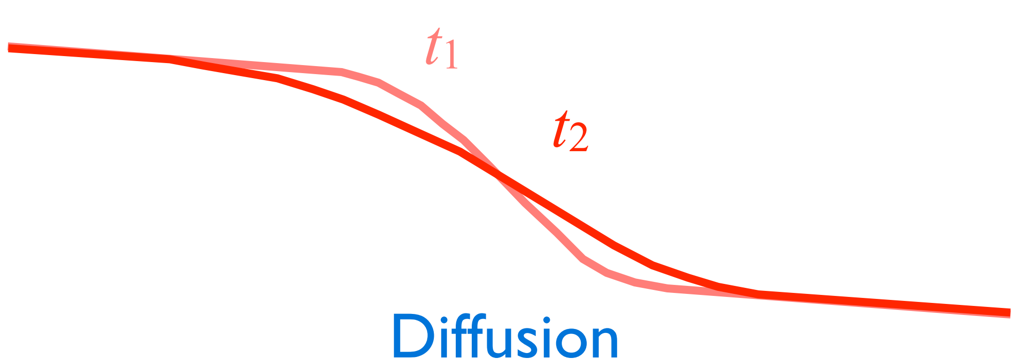

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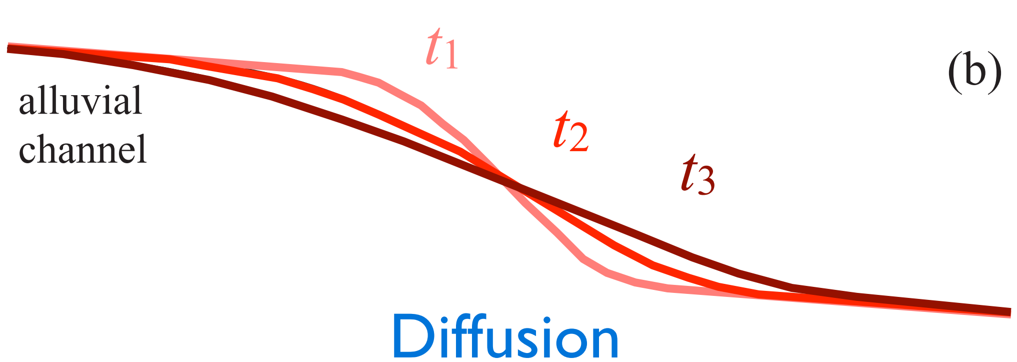

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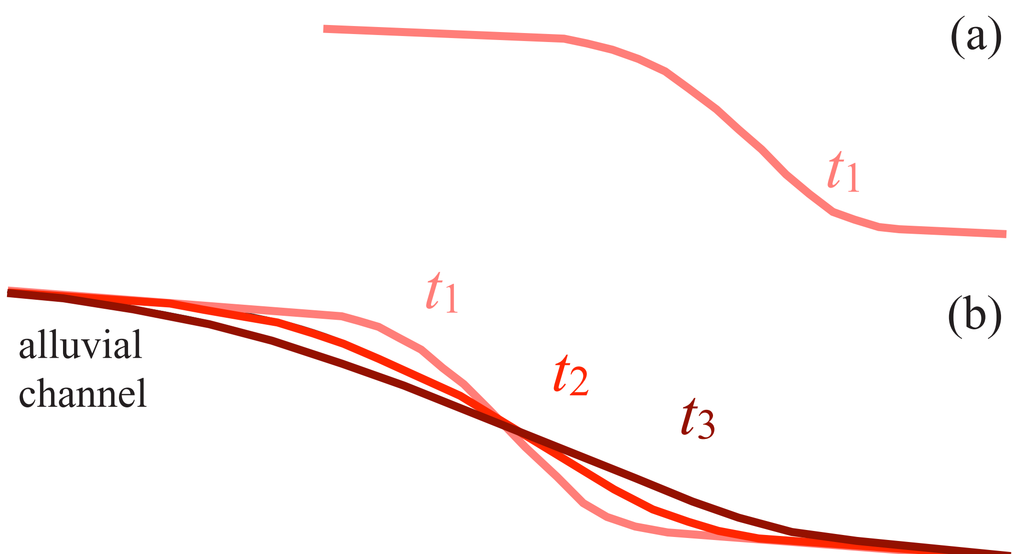

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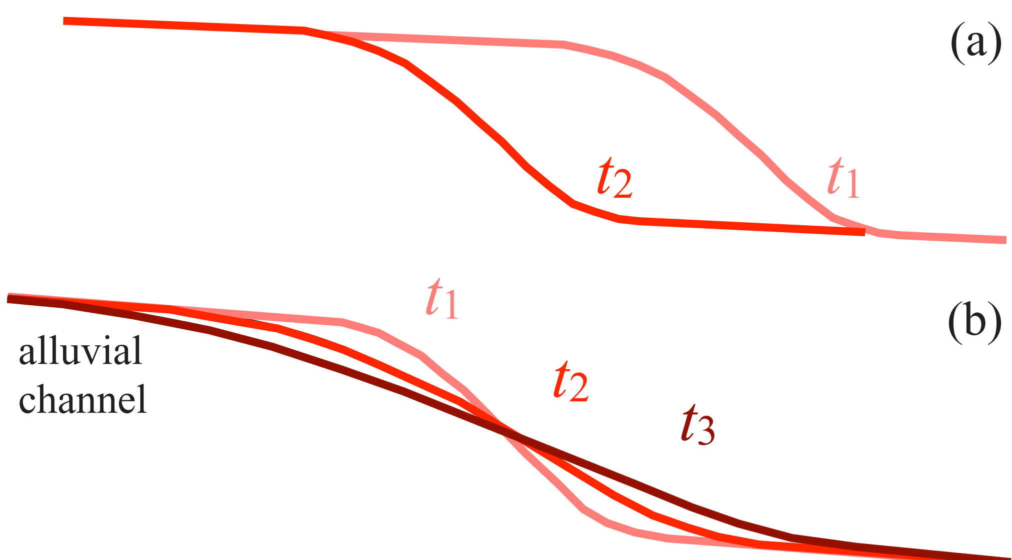

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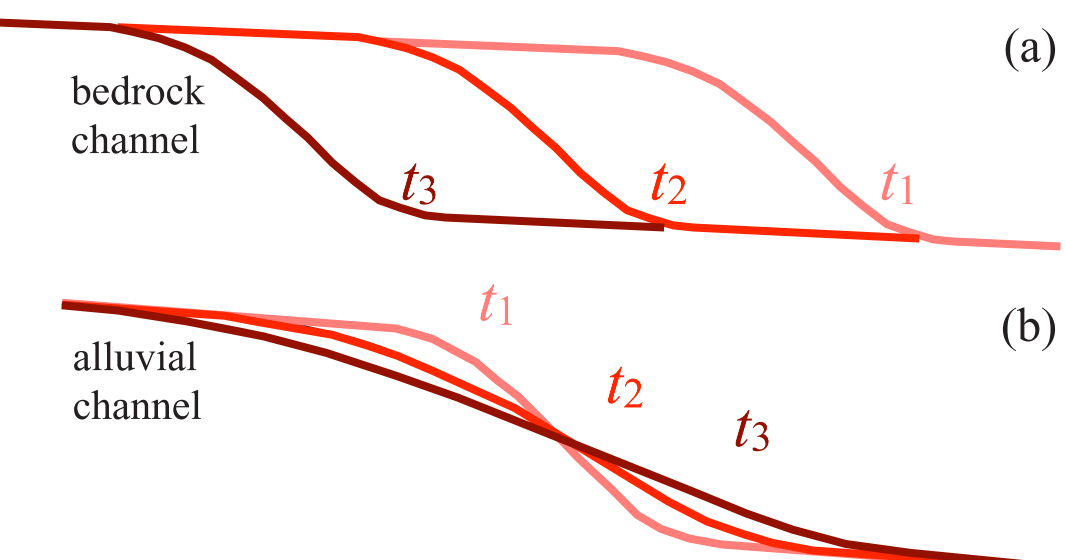

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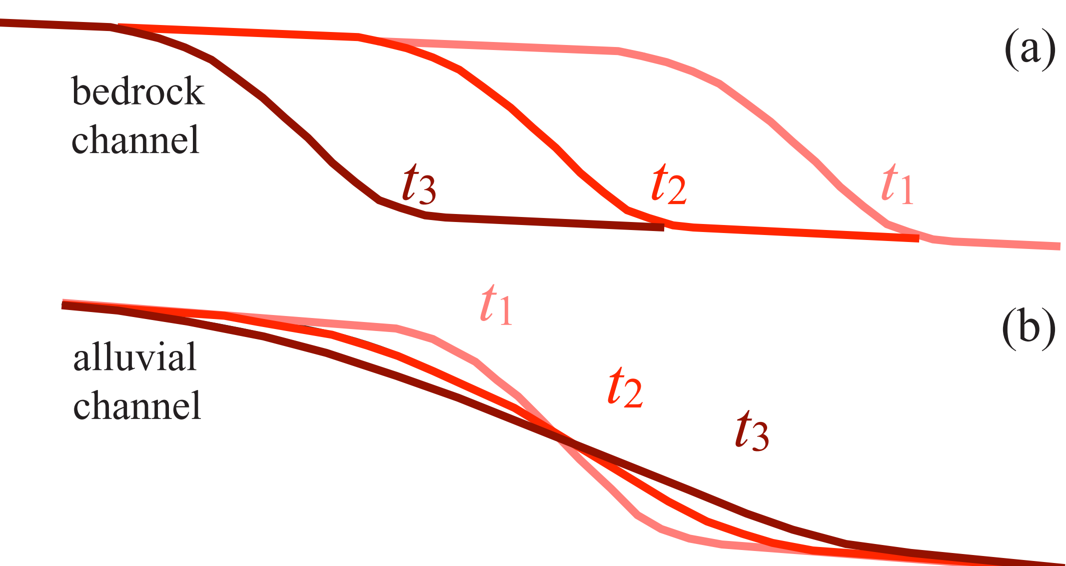

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- **Diffusion**: Rate of erosion depends on change in hillslope gradient (curvature)
- **Advection**: Rate of erosion is directly proportional to hillslope gradient
  - Also, no conservation of mass (deposition)

Recap

- **What are the main differences between the advection and diffusion equations?**

References

Pelletier, J. D. (2008). *Quantitative modeling of earth surface processes* (Vol. 304). Cambridge University Press.