Introduction to Quantitative Geology

Natural diffusion:
Theory and examples

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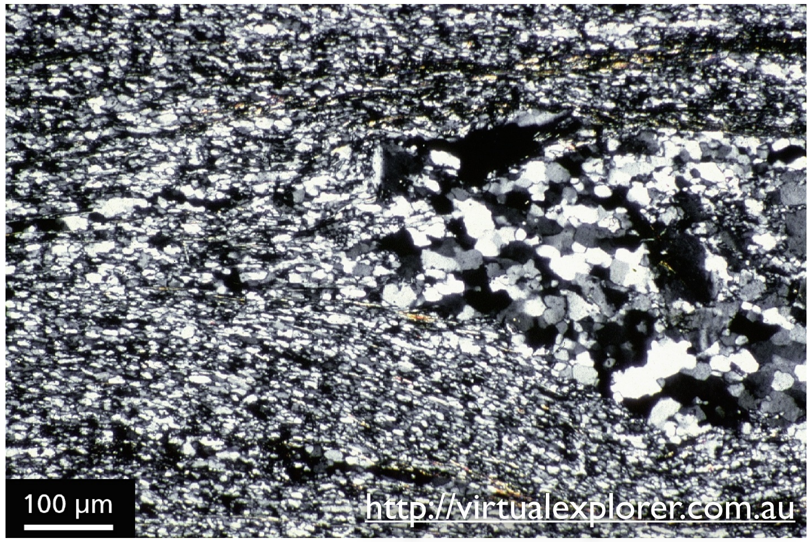
Goals of this lecture

- **The concept**: Diffusion as a process
- **Mathematical definition**: The diffusion equation

The concept

Diffusion as a geological process

Grain boundary sliding

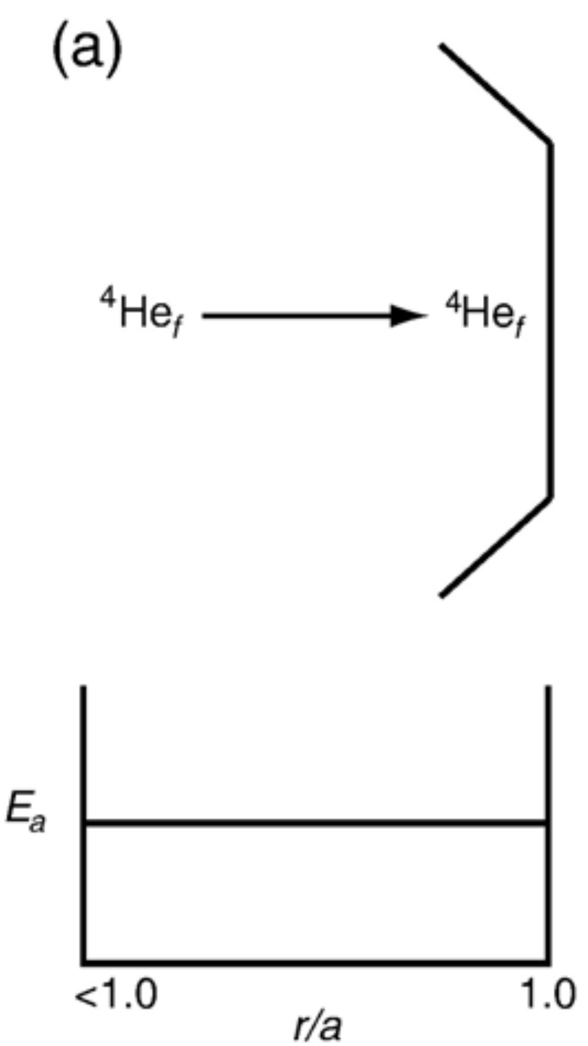


Rock rheology

Rain splash

Hillslope erosion

^{4}He diffusion in apatite



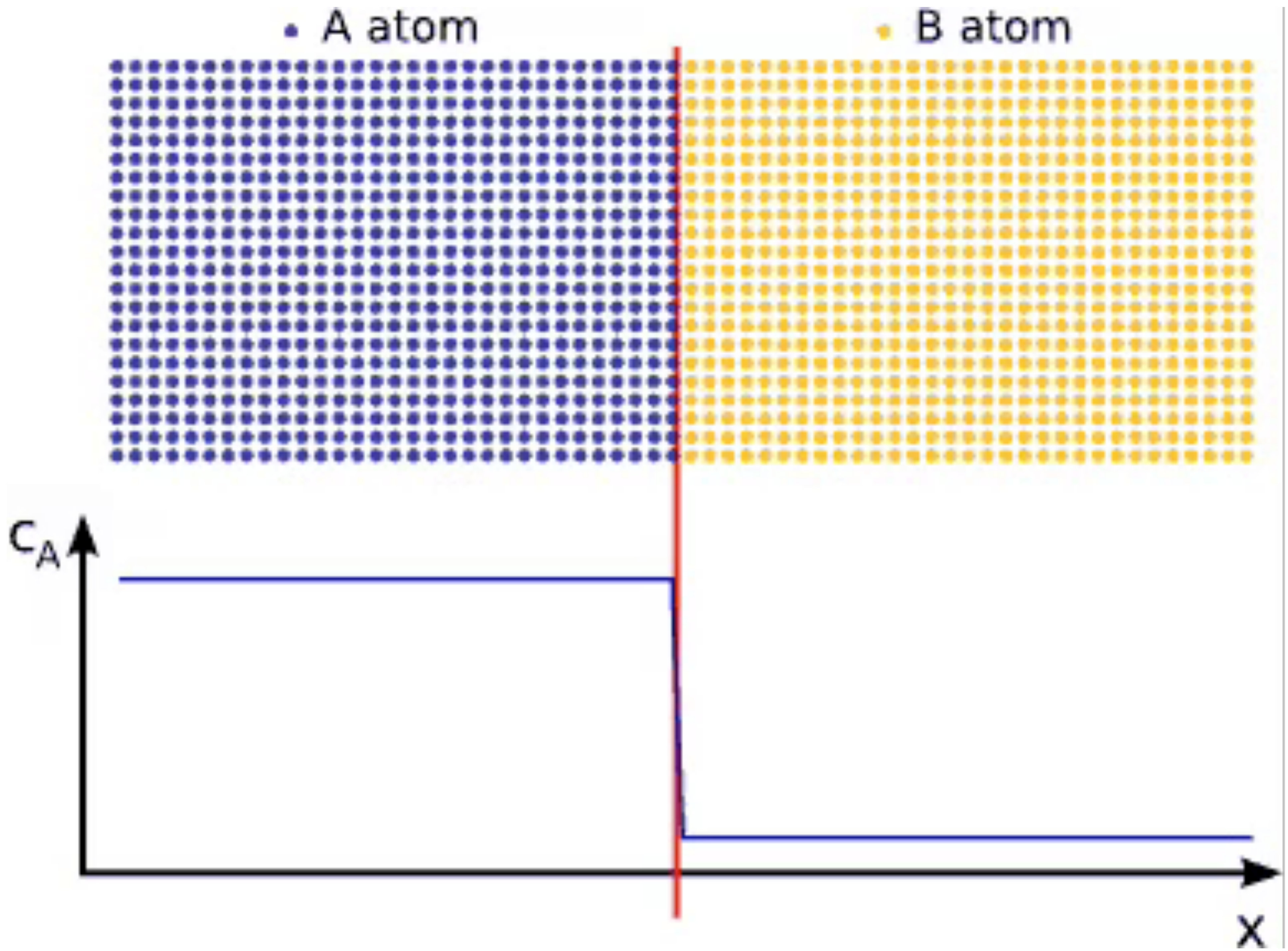
Shuster et al., 2006

Thermochronology

General concepts of diffusion

- **Diffusion** is a process resulting in mass transport or mixing as a result of the random motion of diffusing particles

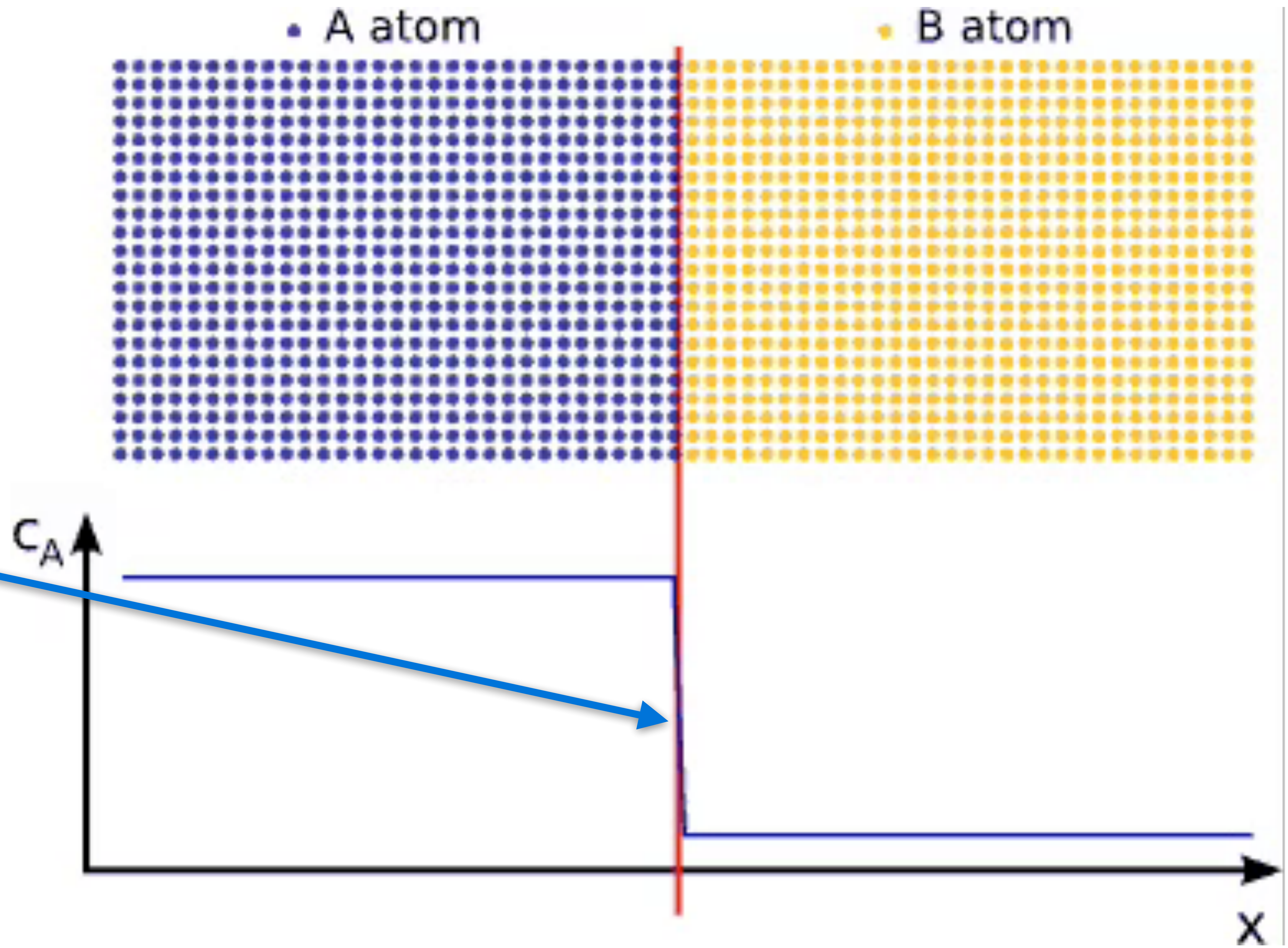
The diffusion process



http://web.unideb.hu/zerdelyi/

The diffusion process

Concentration gradient



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General concepts of diffusion

- **Diffusion** is a process resulting in mass transport or mixing as a result of the random motion of diffusing particles
    - Net motion of mass or transfer of energy is from regions of high concentration to regions of low concentration
    - Diffusion reduces concentration gradients

A more quantitative definition

- **Diffusion** occurs when a **conservative property** moves through space at a **rate proportional to a gradient**

- **Conservative property**: A quantity that must be conserved in the system (e.g., mass, energy, momentum)
- **Rate proportional to a gradient**: Movement occurs in direct relationship to the change in concentration
  - Consider a one hot piece of metal that is put in contact with a cold piece of metal. Along the interface the change in temperature will be most rapid when the temperature difference is largest

Mathematical definition

A mathematical definition

- We can now translate the concept of diffusion into mathematical terms.
    - We've just seen "**Diffusion** occurs when a (1) **conservative property** moves through space at a (2) **rate proportional to a gradient**"
- If we start with part 2, we can say in comfortable terms that
  [transportation rate] is proportional to [change in concentration over some distance]

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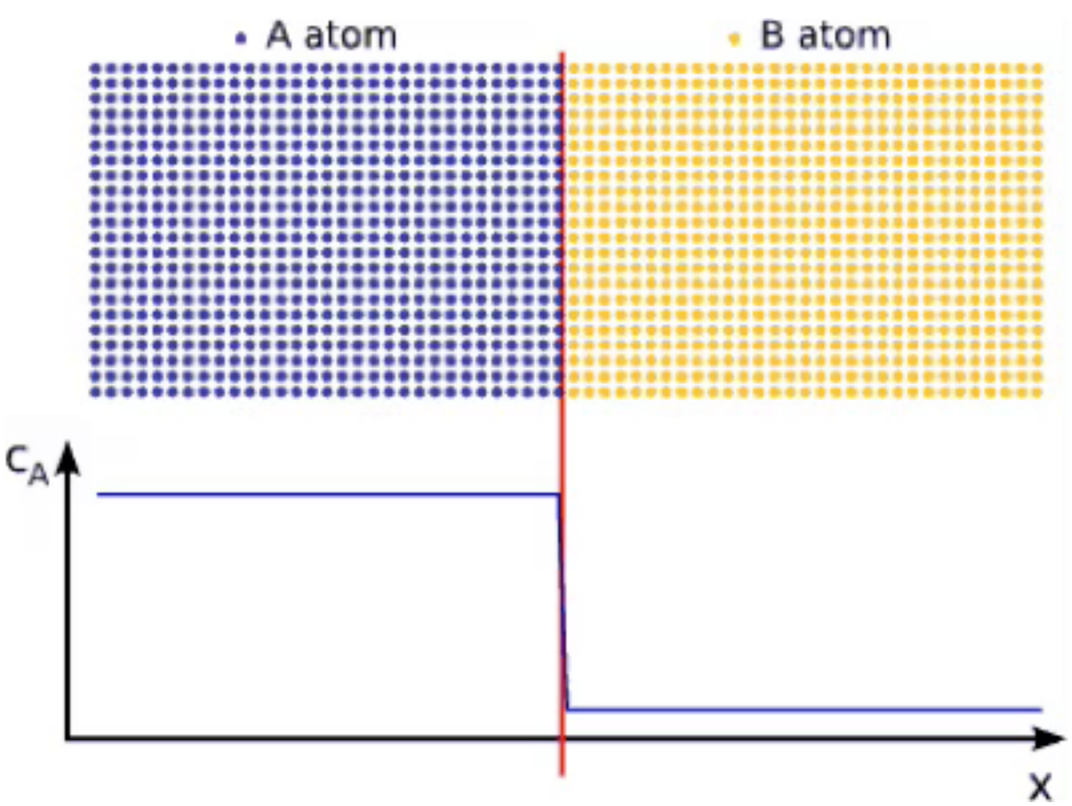
- In slightly more quantitative terms, we could say [flux] is proportional to [concentration gradient]
- Finally, in symbols we can say

$$q \propto \frac{\Delta C}{\Delta x}$$

where q is the mass flux, $\propto$ is the "proportional to" symbol, Δ indicates a change in the symbol that follows, C is the concentration and x is distance

A mathematical definition

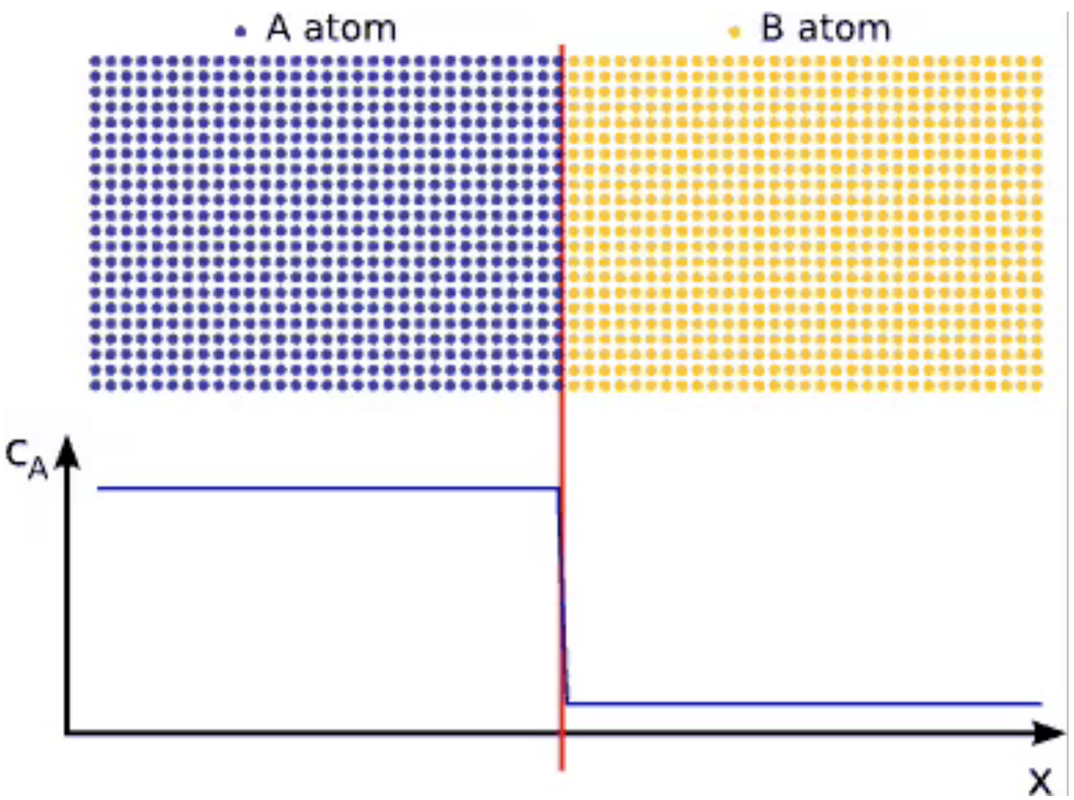


- If transport is directly proportional to the gradient, we can replace the proportional to symbol with a constant
- We can also replace the finite changes Δ with infinitesimal changes ∂
- Keeping the same colour scheme, we see

$$q \propto \frac{\Delta C}{\Delta x} \longrightarrow q = -D \frac{\partial C}{\partial x}$$

where D is a constant called the **diffusion coefficient** or **diffusivity**

A mathematical definition



- Consider the example to the left of the concentration of some atoms A and B
- Here, we can formulate the diffusion of atoms of A across the red line with time as

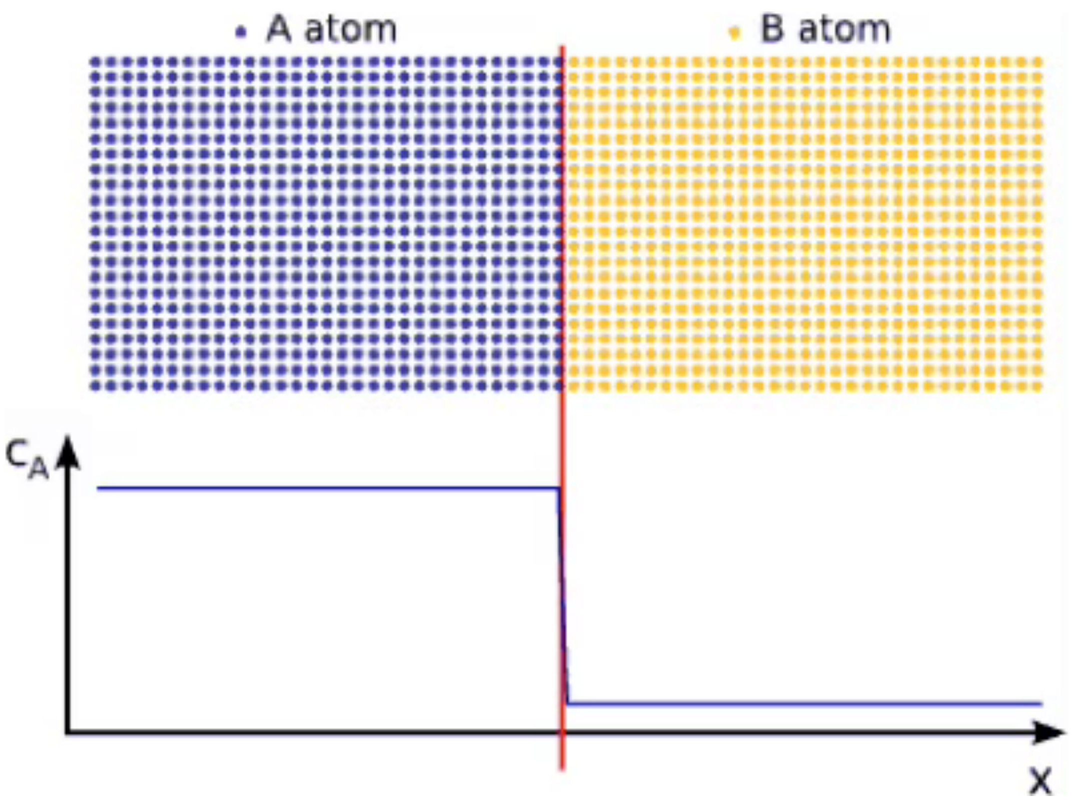
$$q = -D\frac{\partial C_{\mathrm{A}}}{\partial x}$$

where C_{A} is the **concentration** of atoms of A

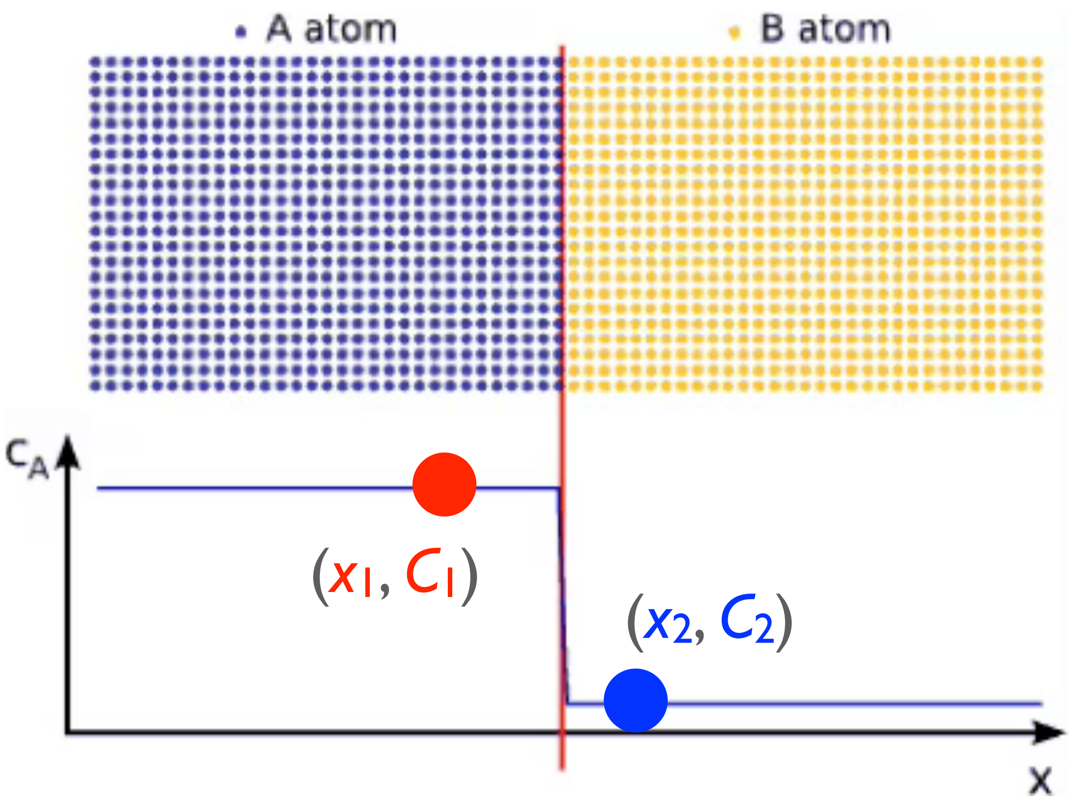
A mathematical definition

- OK, but **why is there a minus sign?**

$$q = -D \frac{\partial C_{\mathrm{A}}}{\partial x}$$



A mathematical definition



- OK, but **why is there a minus sign?**

$$q = -D\frac{\partial C_{\mathrm{A}}}{\partial x}$$

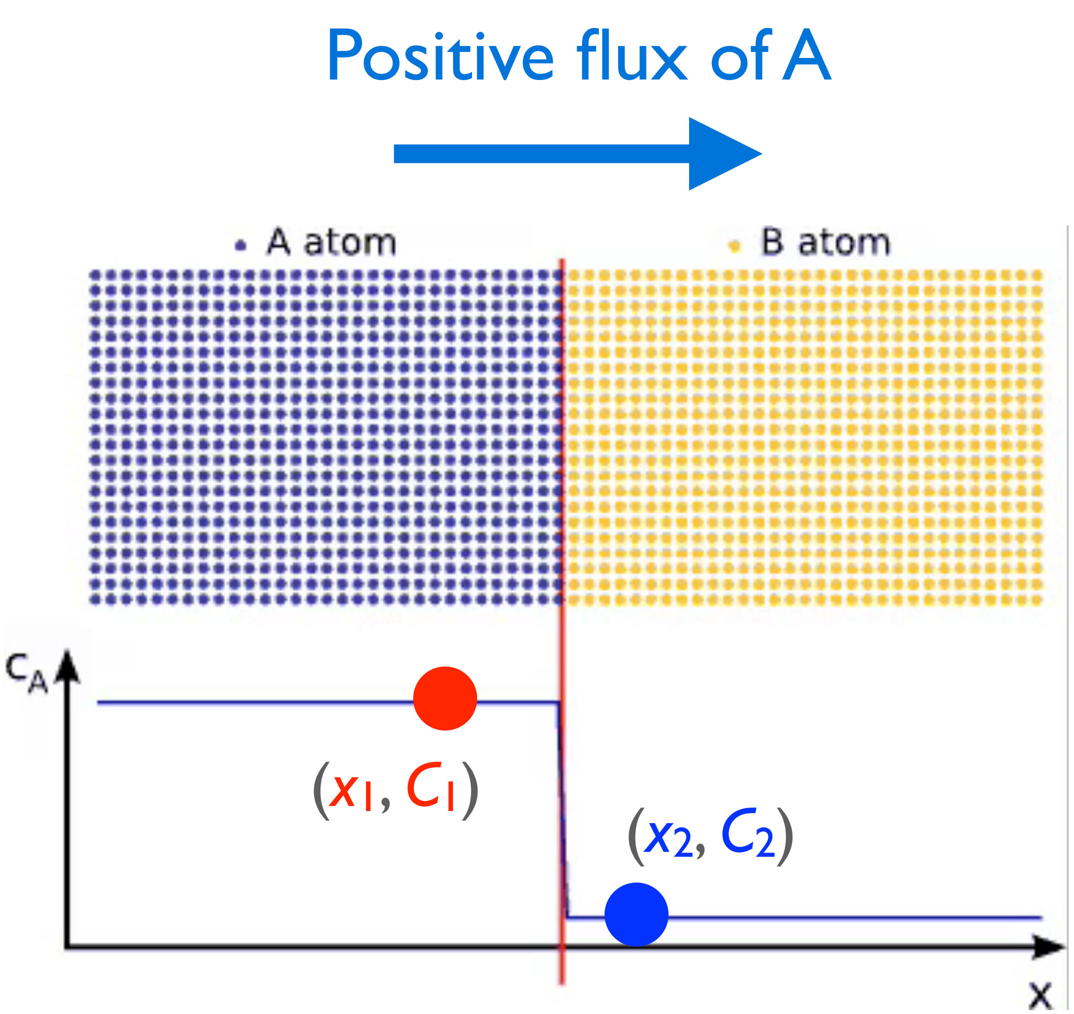
- We can consider a simple case for finite changes at two points: (x_1, C_1) and (x_2, C_2)
- At those points, we could say

$$q = -D\frac{\Delta C}{\Delta x}$$

$$q = -D\frac{C_2 - C_1}{x_2 - x_1}$$

- As you can see, ΔC will be negative while Δx is positive, resulting in a negative gradient

A mathematical definition



- OK, but **why is there a minus sign?**

$$q = -D\frac{\partial C_\mathrm{A}}{\partial x}$$

- Multiplying the negative gradient by **-**D yields a positive flux q along the x axis, which is what we expect

$$q = -D\frac{\Delta C}{\Delta x}$$

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A mathematical definition

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  - We've seen "**Diffusion** occurs when a (1) **conservative property** moves through space at a (2) **rate proportional to a gradient**"
- This part is slightly harder to translate, but we can say that [change in concentration with time] is equal to [change in transport rate with distance]

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$$\frac{\Delta C}{\Delta t} = -\frac{\Delta q}{\Delta x}$$

where **t** is time

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$$\frac{\Delta C}{\Delta t} = -\frac{\Delta q}{\Delta x} \longleftarrow \text{Conservation of mass/energy}$$

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A mathematical definition

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- So, **how is this a conservation of mass/energy equation?**

A mathematical definition

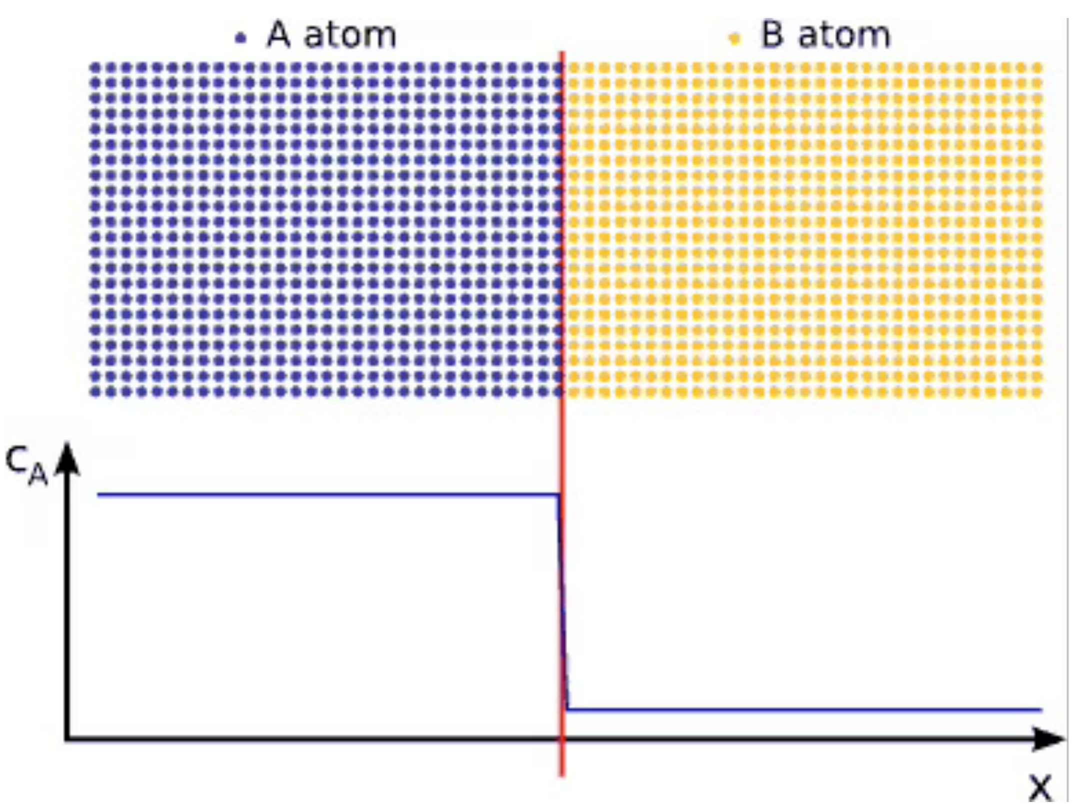
$$\frac{\Delta C}{\Delta t} = -\frac{\Delta q}{\Delta x}$$

- So, **how is this a conservation of mass/energy equation?**

$$\frac{\Delta C}{\Delta t} = -\frac{q_2 - q_1}{x_2 - x_1}$$

- Consider the fluxes q_1 and q_2 at two points, x_1 and x_2
  - What happens when the flux of mass q_2 at x_2 is larger than the flux q_1 at x_1?

A mathematical definition

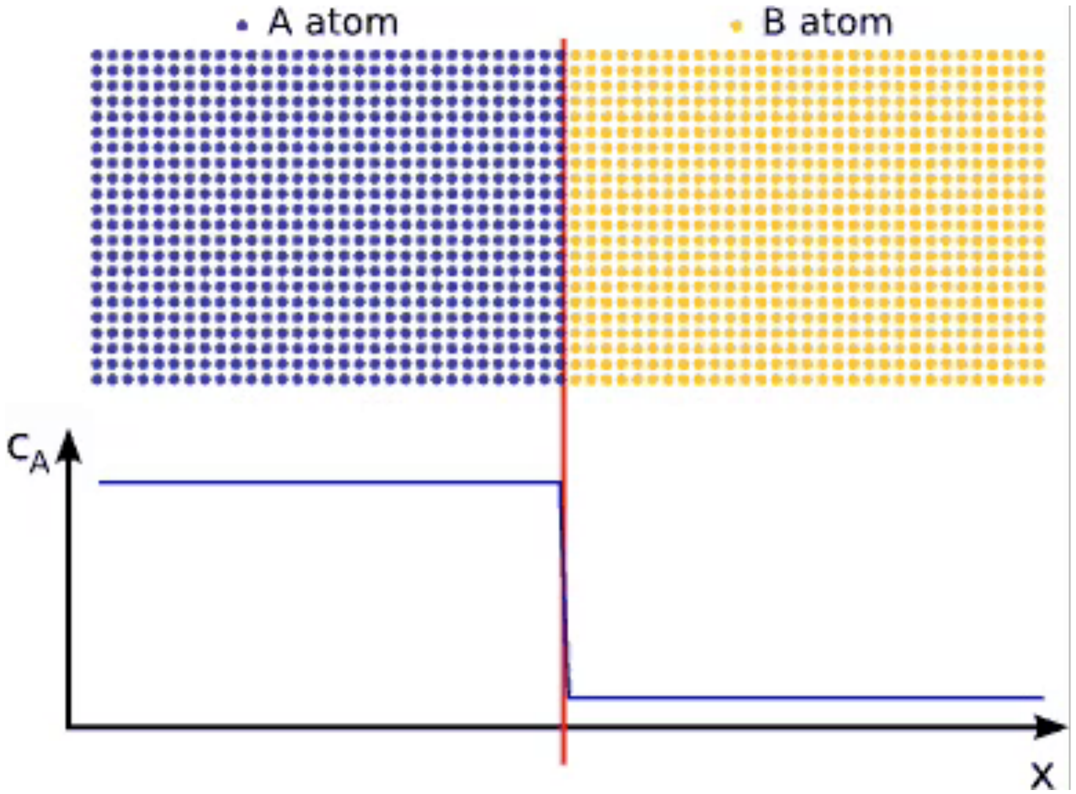


- If we again replace the finite changes Δ with infinitesimal changes ∂, we can describe our example on the left

$$\frac{\partial C_{\mathrm{A}}}{\partial t} = -\frac{\partial q}{\partial x}$$

- Essentially, all this says is that the concentration of A will change based on the flux across a reference face at position x minus the flux across a reference face at position $x + dx$

A mathematical definition



- In the exercise for this week we will explore some simple models for diffusion and some of the factors we need to consider

Up next:
An introduction to thermochronology

- In the next presentation we will explore how we can apply the diffusion equation to determine the age associated with cooling of rocks as they are exhumed toward Earth's surface
  - This method is called thermochronology and we will see it in several examples throughout the rest of the course

Recap

- What are some of the components of diffusion processes?
- What are some geological examples where diffusion might apply?

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- What are some geological examples where diffusion might apply?

References

Shuster, D. L., Flowers, R. M., & Farley, K. A. (2006). The influence of natural radiation damage on helium diffusion kinetics in apatite. *Earth and Planetary Science Letters*, *249*(3-4), 148–161.